8.1

(a) Referring to Fig. 8.2,

note that $I_{D1} = I_{D2} = \frac{I}{2} = \frac{0.2\text{ mA}}{2}$

$= 0.1\text{ mA}$

$$I_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{OV}^2$$

$$V_{ov} = \sqrt{\frac{2I_D}{k_n'(W/L)}} = \sqrt{\frac{2(0.1\text{ mA})}{0.4\text{ mA/V}^2(12.5)}}$$

$= 0.2\text{ V}$

$V_{Gs} = V_{nt} + V_{ov} = 0.5 + 0.2 = 0.7\text{ V}$

(b) If $V_{cm} = 0$,

$V_{s1} = V_{s2} = V_G - V_{GS} = 0 - 0.7 = -0.7\text{ V}$

$I_{D1} = I_{D2} = \frac{0.2\text{ mA}}{2} = 0.1\text{ mA}$

$V_{D1} = V_{D2} = V_{DD} - I_{D1}R_D$

$= 1\text{V} - (0.1\text{ mA})(10\text{ K}) = 0\text{ V}$

(c) Now, if $V_{ICM} = 0.1\text{ V}$,

$V_{S1} = V_{S2} = V_C - V_{GS} = 0.1\text{ V} - 0.7\text{ V}$

$= -0.6\text{V}$

Since I is a constant current source,
I_{D1} and I_{D2} remain at 0.1 mA
This means that
V_{D1} and V_{D2} are still 0 V

(d) $V_{ICM} = -0.1\text{ V}$,

$V_{S1} = V_{S2} = V_G - V_{GS} = -0.1\text{ V} - 0.7$

$= -0.8\text{ V}$

Still, $I_{D1} = I_{D2} = 0.1\text{ mA}$

$V_{D1} = V_{D2} = 0\text{ V}$

(e) $V_{Cm}(\max) = V_{DD} - I_DR_D - V_{ov} + V_{GS}$

$= 1 - (0.1\text{ mA})(10\text{ K}) - 0.2 + 0.7 = +0.5\text{V}$

(f) $V_S(\min) = -V_{SS} + V_{CS}(\min)$

$= -1 + 0.2 = -0.8\text{ V}$

$V_{CM}(\min) = V_S(\min) + V_{GS} = -0.8\text{ V} + 0.7\text{ V}$

$= -0.1\text{ V}$

8.2

$V_{tp} = -0.8\text{ V}\quad k_P'\frac{W}{L} = 4\text{ mA/V}^2$

(a) $V_{G1} = V_{G2} = 0\text{ V}$

$|V_{ov}| = \sqrt{0.5\text{ mA}/4\text{ mA/V}^2} = 0.354\text{ V}$

$|V_{GS}| = |V_{tp}| + |V_{ov}| = 1.154\text{ V}$

$V_S = V_{G1} + |V_{GS}| = 1.154\text{ V}$

$$V_{D1} = V_{D2} = -2.5\text{ V} + \left(\frac{0.5\text{ mA}}{2}\right)(4\text{ k}\Omega)$$

$= -1.5\text{ V}$

(b) Current source requires 0.5 V

$V_{CM(\max)} = 2.5\text{ V} - 0.5\text{ V} - 0.8\text{ V} - 0.354\text{ V}$

$= 0.846\text{ V}$

$$V_{CM(\min)} = -2.5\text{ V} + \left(\frac{0.5\text{ mA}}{2}\right)(4\text{ k}\Omega) - 0.8\text{ V}$$

$= -2.3\text{ V}$

8.3

Refer to Fig.8.2

$$V_{OV} = \sqrt{\frac{2I_D}{k_n'(W/L)}} = \sqrt{\frac{2(0.1\text{ mA})}{0.4\text{mA}/V^2(12.5)}} = 0.2\text{ V}$$

(a) $V_{GS} = V_{OV} + V_t = 0.2\text{ V} + 0.5\text{ V} = 0.7\text{ V}$

$V_S = V_G - V_{GS} = 0 - 0.7\text{ V} = -0.7\text{ V}$

$V_{D1} = V_{D2} = V_{DD} - i_{D1}R_D = 1.0\text{ V} - 0.1\text{ mA}$
$(10\text{ k}\Omega) = 0\text{ V}$

$V_{D2} - V_{D1} = 0\text{ V}$

(b) For $i_{D1} = 0.15\text{ mA}$, $i_{D2} = 0.05\text{ mA}$,

$$i_{D1} = \frac{I}{2} + \frac{I}{V_{OV}}\cdot\frac{v_{id}}{2} \rightarrow v_{id} = \left(\frac{2i_{D1}}{I} - 1\right)\cdot V_{OV}$$

$$v_{id} = \left[\frac{2(0.15\text{ mA})}{0.2\text{ mA}} - 1\right](0.2\text{ V}) = 0.1\text{ V}$$

$$V_{GS1} = \sqrt{\frac{2(0.15\text{ mA})}{0.4\text{ mA/V}^2(12.5)}} + 0.5\text{V} = 0.745\text{ V}$$

$V_S = V_G - V_{GS1} = 0.1\text{ V} - 0.745\text{ V} = -0.645\text{ V}$

$V_{D1} = V_{DD} - i_{D1}R_D = 1.0\text{ V} - 0.15\text{ mA}(10\text{ k}\Omega)$

$= -0.5\text{ V}$

$V_{D2} = 1.0\text{ V} - (0.05\text{ mA})(10\text{ k}\Omega) = +0.5\text{ V}$

$V_{D2} - V_{D1} = 1.0\text{ V}$

(c) $i_{D1} = 0.2\text{ mA},\ i_{D2} = 0:$

$V_{G1} = v_{id} = \sqrt{2}\cdot V_{OV} = 1.414(0.2\text{ V}) = 0.283\text{ V}$

$$V_{GS} = \sqrt{\frac{2(0.2\text{ mA})}{0.4\text{ mA/V}^2(12.5)}} + 0.5\text{V} = 0.783$$

$V_S = V_G - V_{GS} = 0.283\text{V} - 0.783\text{V} = -0.5\text{V}$

$V_{D1} = 1.0\text{ V} - (0.2\text{ mA})(10\text{ k}\Omega) = -1.0\text{ V}$

$V_{D2} = +1.0\text{ V}$

$V_{D2} - V_{D1} = 2.0\text{ V}$

(d) $\left.\begin{array}{l} i_{D1} = 0.05\text{ mA} \\ i_{D2} = 0.05\text{ mA}\end{array}\right\}$ opposite case of (b)

For example,

$$v_{id} = \left[\frac{2(0.05\text{ mA})}{0.2\text{ mA}} - 1\right](0.2\text{ V}) = -0.1\text{ V}$$

$$V_{GS} = \sqrt{\frac{2(0.05\text{ mA})}{0.4\text{ mA/V}^2(12.5)}} + 0.5\text{ V} = 0.641\text{ V}$$

$V_S = V_G - V_{GS} = -0.1\text{ V} - 0.641\text{V} = -0.741\text{ V}$

$V_{D1} = 1.0\text{ V} - (0.05\text{ mA})(10\text{ k}\Omega) = +0.5\text{ V}$

$V_{D2} = 1.0\text{ V} - (0.05\text{ mA})(10\text{ k}\Omega) = -0.5\text{ V}$

$V_{D2} - V_{D1} = -1.0\text{ V}$

(e) $i_{D1} = 0\text{ mA},\ i_{D2} = 0.2\text{ mA}$ is the opposite of (c):

$v_{id} = -\sqrt{2}(V_{OV}) = -\sqrt{2}(0.2\text{ V}) = -0.283\text{ V}$

For $i_{D2} = 0.2\text{ mA},\ V_{GS2} = 0.783\text{ V}$, So that

$V_S = -0.783\text{ V}$

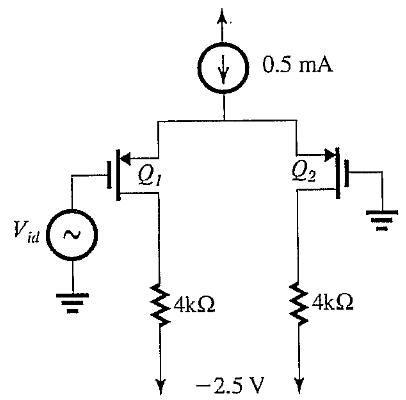
$V_{D1} = 1.0\text{ V},$

$V_{D2} = -1.0\text{ V} \rightarrow V_{D2} - V_{D1} = -2\text{ V}$

The results are summarized in the following table:

Case	V_{id} *(V)*	i_{D1} *(mA)*	i_{D2} *(mA)*	V_S*(V)*	V_{D1}*(V)*	V_{D2} *(V)*	$V_{D2} - V_{D1}$*(V)*
(a)	0	0.1	0.1	−0.7	0	0	0
(b)	0.1	0.15	0.05	−0.645	–0.5	0.5	1.0
(c)	0.283	0.2	0	–0.5	–1.0	1.0	2.0
(d)	–0.1	0.05	0.15	–0.741	0.5	–0.5	–1.0
(e)	–0.283	0	0.2	–0.783	1.0	–1.0	–2.0

8.4



$V_{G2} = 0$

$V_{G1} = v_{id}$

When all the current is on Q_1:

$$I = \frac{1}{2}\left(k_p'\frac{W}{L}\right)(V_{GS1} - V_t)^2$$

$$\Rightarrow V_{GS1} = V_t + \sqrt{\frac{2I}{k_p' w/L}}$$

$= V_t + \sqrt{2}V_{OV}$

and V_{GS2} is reduced to V_t thus $V_S = -V_t$.

Then $v_{id} = v_{GS1} + v_S$

$= V_t + \sqrt{2}\,V_{OV} - V_t = \sqrt{2}\,V_{ov}$

In a similar manner as for the NMOS Differential Amplifier, as v_i reaches $-\sqrt{2}\,V_{OV}$ Q_1 turns off and Q_2 on. Thus the steerine range is

$\sqrt{2}\,V_{OV} \le V_i \le -\sqrt{2}\,V_{OV}$

For this particular case

$$V_{ov} = \sqrt{\frac{0.25\text{ mA}}{4\text{ mA/V}^2}} = 0.25\text{ V}$$

$\sqrt{2} \times -0.25 \le v_{id} \le \sqrt{2} \times 0.25$

$-0.35 \le v_{id} \le 0.35$

when $V_{id} = -0.35$ V,

$i_{D1} = 0.5\text{ mA},\ i_{D2} = 0$

$V_S = -V_{t2} = +0.8\text{ V}$

$V_{D1} = 4\text{ k}\Omega \times 0.5\text{ mA} - 2.5 = -0.5\text{ V}$

$V_{D2} = 0 - 2.5\text{ V} = -2.5\text{ V}$

when $v_{id} = +0.35$ V,

$i_{D1} = 0\ ;\ i_{D2} = 0.5\text{ mA}$

$V_S = v_{id} - v_{GS1} = v_{id} - V_{t1}$

$= 0.35\text{ V} + 0.8\text{ V} = 1.15\text{ V}$

$V_{D1} = -2.5\text{ V}$

$V_{D2} = -0.5\text{ V}$

8.5

$V_{G1} = v_{id}\ i_{D1} = 0.11\text{ mA}$

$V_{G2} = 0\quad i_{D2} = 0.09\text{ mA}$

$$I_D = \frac{1}{2}k_n'\frac{W}{L}(V_{GS} - V_t)^2$$

For Q_1:

$0.11\text{ m} = \frac{1}{2}5\text{ m}(V_{GS1} - 0.5)^2$

$\rightarrow V_{GS1} = 0.71\text{ V}$

For Q_2:

$0.09\text{ m} = \frac{1}{2}5\text{ m}(V_{GS2} - 0.5)^2$

$\rightarrow V_{GS2} = 0.69\text{ V}$

$V_S = -V_{GS2} = -0.69\text{ V}$

$v_{id} = V_S + V_{GS1} = -0.69 + 0.71$

$= 0.02\text{ V}$

$V_{D2} - V_{D1} = 10\ \text{k}\Omega\ (i_{D1} - i_{D2})$
$= 10$ kV (0.11 2 0.09) m
$= 0.2$ V
thus

$$\frac{V_{D2} - V_{D1}}{v_{id}} = \frac{0.2}{0.02} = 10$$

when $i_{D1} = 0.09$ mA and

$i_{D2} = 0.11$ mA

is the reverse condition from the case we just studied, thus $v_{id} = -0.02$ V

8.6

$V_{GS} = V_{tn} + V_{OV} = 0.5\text{ V} + 0.2\text{ V} = 0.7\text{ V}$

$V_{D4} = V_{G4} = -V_{SS} + V_{GS} = -1.2\text{ V} + 0.7\text{ V}$

$= -0.5$ V

$$R = \frac{V_{DD} - V_{D4}}{0.1\text{ mA}} = \frac{1.2\text{V} - (-0.5\text{ V})}{0.1\text{ mA}} = 17\text{ k}\Omega$$

$$R_D = \frac{V_{DD} - V_{D1}}{0.4\text{ mA}/2} = \frac{1.2\text{V} - 0.2\text{ V}}{0.2\text{ mA}} = 5\text{ k}\Omega$$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{0.4\text{ mA}}{2}\left[k_n' V_{ov}^2\right]$$

$$= 0.2\text{ mA}[(0.25\text{ mA}/\text{V}^2)(0.2\text{V})^2]^{-1} = 20$$

$$\left(\frac{W}{L}\right)_3 = 0.4\text{ mA}[0.01\text{ mA}]^{-1} = 40$$

$$\left(\frac{W}{L}\right)_4 = 0.1\text{ mA}[0.01\text{ mA}]^{-1} = 10$$

$V_{Cm(\max)} = V_{tn} + V_{DD} - (I/2)R_D$

$= 0.5\text{ V} + 1.2\text{ V} - (0.2\text{ mA})(5\text{ k}\Omega) = 0.7\text{ V}$

$V_{Cm(\min)} = -V_{SS} + V_{OV3} + V_{tn} + V_{OV1}$

$= -1.2\text{ V} + 0.2\text{ V} + 0.5\text{ V} + 0.2\text{ V} = -0.3\text{ V}$

8.7

$|V_{id}|_{MAX} = 160$ mV

$$\left(\frac{V_{id}/2}{V_{OV}}\right)^2 = 0.1$$

$$V_{OV}^2 = \frac{(80\text{ mV})^2}{0.1}$$

$$V_{OV} = \sqrt{\frac{(80\text{ mV})^2}{0.1}} = 253\text{ mV}$$

$I = 0.4$ mA

$k_n' = 0.2\text{ mA}/\text{V}^2$

$$\frac{W}{L} = I \cdot \left[k_n' V_{ov}^2\right]^{-1}$$

$$= 0.4 \cdot [(0.2)(0.253)^2]^{-1} = 31.2$$

$$g_m = \frac{I}{V_{OV}} = \frac{0.4\text{ mA}}{0.253\text{ V}} = 1.58\text{ mA}/\text{V}$$

8.8

$$\left(\frac{v_{id\max}/2}{V_{OV}}\right)^2 = \text{K}$$

$$\Rightarrow 2V_{OV}\sqrt{\text{K}} = v_{id\max}$$

Q.E.D.

$$i_{D1} = \frac{I}{2} + \left(\frac{I}{V_{OV}}\right)\frac{v_{id}}{2}\sqrt{1-K}$$

$$i_{D1} = \frac{I}{2} \pm \frac{I}{V_{OV}} \cdot \frac{2}{2}V_{OV}\sqrt{K} \cdot \sqrt{1-K}$$

$$\rightarrow i_{D1} = \frac{I}{2} \pm I\sqrt{K(1-K)}$$

thus $\Delta I = 2I\sqrt{K(1-K)}$

Q.E.D.

For K = 0.01

$\Delta I = 2I\sqrt{0.01(1-0.01)}$

$= 0.198 \times I$

$V_{id\max} = 2V_{OV}\sqrt{0.01} = 0.2V_{OV}$

For K = 0.1

$\Delta I = 2I\sqrt{0.1(1-0.1)} = 0.8I$

$V_{id\max} = 2V_{OV}\sqrt{0.2}$

$= 0.894.\ V_{OV}$

8.9

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{tn})^2$$

$$\frac{400}{2} = \frac{1}{2}(200)\left(\frac{20}{0.5}\right)(V_{GS} - 0.5)^2$$

$(V_{GS} - 0.5)^2 = 1/20$ V

$V_{GS} = 0.724$ V

$$g_m = \frac{2I_D}{V_{GS} - V_{tn}} = \frac{2 \cdot 400\ \mu\text{A}}{0.724\text{ V} - 0.5\text{ V}}$$

$= 3.57$ mA/V

V_{id} for full current switching

$= \sqrt{2}\ (V_{GS} - V_{tn}) = 0.317$ V

To double this value, V_{OV}, so quadruple I_D to 1.6 mA

8.10

$V_{OV} = 0.25\text{V}$ $g_m = 1\text{ mA}/\text{V}$ $V_{tn} = 0.8$ V

$k_n' = 100\ \mu\text{A}/\text{V}^2$

$I = g_m V_{OV} = 0.25$ mA

$$I_D = \frac{I}{2} = \frac{1}{2}k_n'\frac{W}{L}V_{OV}^2$$

$$\frac{W}{L} = \text{I}/\left(k_n' V_{OV}^2\right)$$

$= 0.25\text{ mA}/(0.1\text{ mA}/\text{V}^2 \cdot (0.25\text{ V})^2)$

$= 0.25\text{ mA}/0.00625\text{ mA} = 40$

8.11

$$i_D = \frac{1}{2}k_n\frac{W}{L}(V_{GS} - V_t)^2$$

$$50 = \frac{1}{2}\times 400(V_{GS} - 1)^2$$

$$\Rightarrow V_{GS} = 1.5\text{ V}$$

For $v_{GI} = v_{G2} = 0$, $v_S = -1.5\text{ V}$

For $v_{GI} = v_{G2} = 2\text{ V}$, $v_S = +0.5\text{ V}$

The drain currents are equal in both cases.

For $V_{G2} = 0$:

To reduce i_{D2} by 10%,

$$i_{D2} = 0.9\times 50 = 45\ \mu\text{A}$$

$$i_{D1} = 55\ \mu\text{A}$$

$$v_{GS2} = \sqrt{\frac{2i_{D2}}{400}} + 1 = 1.47\text{ V}$$

$$v_{GS1} = \sqrt{\frac{2\times 55}{400}} + 1 = 1.52\text{ V}$$

Thus, $V_{G1} \equiv v_{GS1} - v_{GS2} = 0.05\text{ V}$

To increase i_{D2} by 10%

$$i_{D2} = 55\ \mu\text{A}$$

$$i_{D1} = 45\ \mu\text{A}$$

$$v_{GS2} = 1.52\text{ V}$$

$$v_{GS1} = 1.47\text{ V}$$

$$\Rightarrow V_{G1} = -0.05\text{ V}$$

i_{D2}/i_{D1}	i_{D2} (μA)	i_{DI} (μA)	V_{GS2} (V)	V_{GSI} (V)	$V_G - V_{GI}$ (V)
1	50	50	1.5	1.5	0
0.5	33.3	66.7	1.408	1.577	−0.17
0.8	47.4	52.6	1.487	1.513	−0.026
0.99	47.75	50.25	1.4886	1.5012	−0.013

For $i_{D1}/i_{D2} = 20 \Rightarrow i_{D2} = 4.76\ \mu\text{A}$

$i_{D1} = 95.24\ \mu\text{A}$

$V_{GS2} = 1.154\text{ V}$, $V_{GS1} = 1.690$

Thus $V_{G1} - V_{G2} = 0.536\text{ V}$

8.12

(a) $V_{od} = V_{D2} - V_{D1} =$

$(V_{DD} - i_{D2}R_D) - (V_{DD} - i_{D1}R_D) = (i_{D1} - i_{D2})R_D$

USING 8.23 AND 8.24

$$V_{od} = \left[\left(\frac{I}{V_{OV}}\right)\left(\frac{V_{id}}{2}\right)\sqrt{1 - \left(\frac{V_{id}/2}{V_{OV}}\right)^2} + \left(\frac{I}{V_{OV}}\right)\left(\frac{V_{id}}{2}\right)\sqrt{1 - \left(\frac{V_{id}/2}{V_{OV}}\right)^2}\right]R_D$$

$$= IR_D\frac{V_{id}}{V_{OV}}\sqrt{1 - \left(\frac{V_{id}/2}{V_{OV}}\right)^2}$$

(b) see plot

slope of linear portion

$$= \frac{d}{dV_{id}}\left(\frac{IR_D}{V_{OV}}\cdot V_{id}\right) = IR_D/V_{OV}$$

(c) see plot

when the bias current is doubled, V_{OV} so

$$V_{od}/V_{id} = \frac{2IR_D}{\sqrt{2}\,V_{OV}}\sqrt{1 - \left(\frac{V_{id}/2}{\sqrt{2}\,V_{OV}}\right)^2}$$

increases by a factor of $\sqrt{2}$ the slope of the linear part has increased by a factor of $\sqrt{2}$

(d) see plot

If W/L is doubled, V_{OV} reduces by a factor at $\sqrt{2}$

$$\text{so } V_{od}/V_{id} = \frac{2IR_D}{\sqrt{2}\,V_{OV}}\sqrt{1 - \left(\frac{V_{id}/\sqrt{2}}{V_{OV}}\right)^2}$$

The slope of the linear part has increased by factor of $\sqrt{2}$ compared to (b)

8.13

$I = 0.4\text{ mA}$ $W/L = 32$ $k_n' = \mu_n C_{ox}$

$= 200\ \mu\text{A/V}^2$

$V_A = 10\text{ V}$ $R_D = 5\text{ k}\Omega$

$$V_{OV}' = \sqrt{I/k_n'\left(\frac{W}{L}\right)} = \sqrt{0.4/(0.2\cdot 32)}$$

$$= 0.25\text{ V}$$

$$g_m = \frac{I}{V_{OV}} = \frac{0.4\text{ mA}}{0.25\text{ V}} = 1.6\text{ mA/V}$$

$$r_O = \frac{V_A}{I_D} = \frac{10\text{ V}}{0.2\text{ mA}} = 50\text{ k}\Omega$$

$$A_d = g_m(R_D \| r_O) = 1.6(5\|50)$$

$$= 1.6(4.54) = 7.3\text{ V/V}$$

8.14

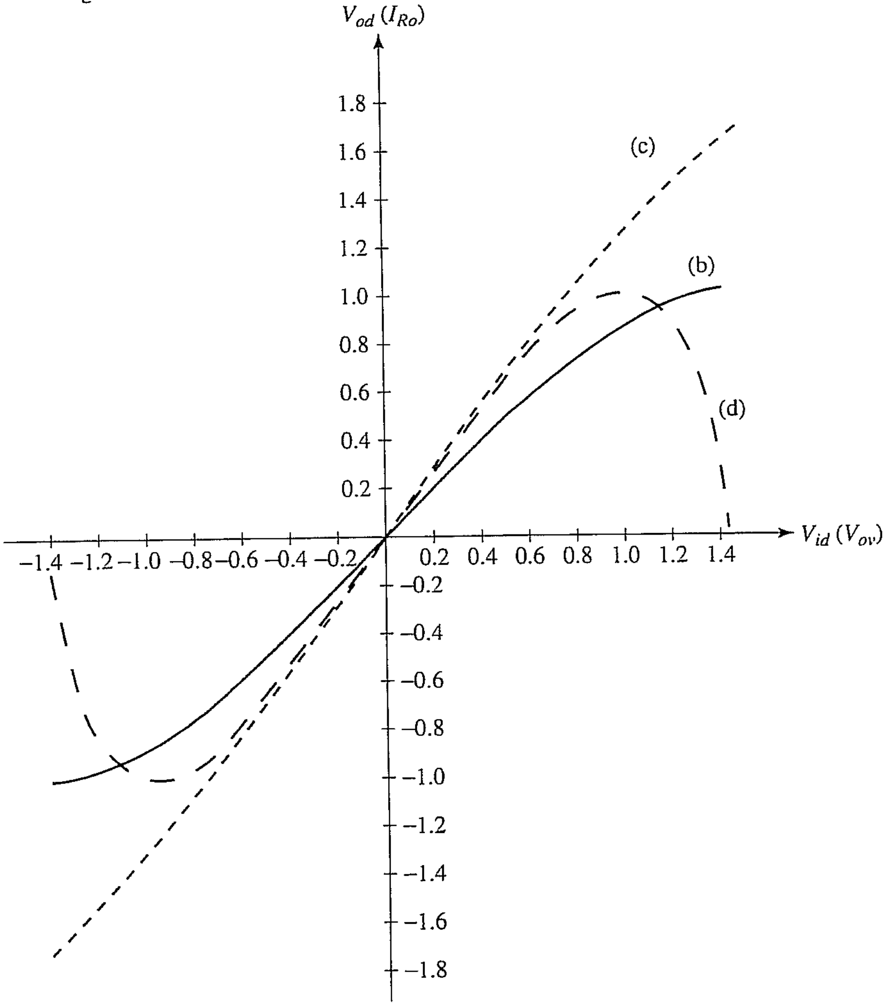
$$\left(\frac{V_{id}/2}{V_{OV}}\right)^2 = 0.05$$

$$\left(\frac{0.1/2}{V_{OV}}\right) = \sqrt{0.05}$$

$$V_{OV} = \frac{0.05}{\sqrt{0.05}} = 0.224\text{ V}$$

$$g_m = \frac{I}{V_{OV}}$$

The figure belongs to 8.12



$I = g_m V_{OV} = (1 \text{ mA/V})(0.224 \text{ V})$

$= 0.224 \text{ mA}$

$A_d = g_m R_D = (1 \text{ mA/V})(10 \text{ k}\Omega) = 10$

$V_{od} = A_d V_{id} = (10)(0.1 \text{ V}) = 1 \text{ V}$

$\frac{W}{L} = \text{I}/\left(k_n' V_{OV}^2\right)$

$= 0.224/(0.2 \times 0.224^2)$

$= 22.3$

$\frac{W}{L} = 2(I/2)/(k_n' V_{OV}^2)$

$= 1 \text{ mA}/(0.4 \text{ mA/V}^2 \cdot (0.2 \text{ V})^2) = 62.5$

BECAUSE WE PICKED I = 1 mA THIS IS THE SOLUTION WITH THE HIGHEST ALLOWABLE POWER. THIS SOLUTION WILL ALSO THEREFORE HAVE THE WIDEST RANGE OF DIFFERENTIAL MODE OPERATION. AN INFINITE NUMBER OF OTHER SOLUTIONS EXIST.

8.15

$+1$ V supplies not more than 2 mW $A_d = 5$ V/V

$V_D = 0.5$ V $K_n' = \mu_n C_{ox} = 0.4 \text{ mA/V}^2$

$$I = \frac{2 \text{ mW}}{1 \text{ V} - (-1 \text{ V})} = 1 \text{ mA}$$

$$R_D = \frac{1 \text{ V} - 0.5 \text{ V}}{1/2\ I} = \frac{0.5 \text{ V}}{0.5 \text{ mA}} = 1 \text{ k}\Omega$$

$$g_m = \frac{A_d}{R_D} = \frac{5 \text{ V/V}}{1 \text{ k}\Omega} = 5 \text{ mA/V}$$

$$V_{OV} = \frac{I}{g_m} = \frac{1 \text{ mA}}{5 \text{ mA/V}} = 0.2 \text{ V}$$

8.16

$$I = \frac{2 \text{ mW}}{1\text{V} - (-1 \text{ V})} = 1 \text{ mA}$$

$0.4 \text{ V} = 2\sqrt{2}\, V_{OV} \quad V_{OV} = 0.141 \text{ V}$

$$R_D = A_d \frac{V_{OV}}{I} = 5^{\text{V/V}}\left(\frac{0.141 \text{ V}}{1 \text{ mA}}\right) = 705\ \Omega$$

$$\frac{W}{L} = \text{I}/\left(k_n' V_{ov}^2\right) = 125$$

since $A_d = g_{m1,2}(r_{O1} \| R_O')$,

$$A_d = g_{m1,2}\left[r_{O1,2} \| \left(\frac{r_{O3,4}}{1 + g_{m3,4}r_{O3,4}}\right)\right]$$

(b) If r_O is ignored,

$$R_D' = \frac{1}{\frac{1}{r_O} + g_m} \simeq \frac{1}{g_m}$$

so that

$$A_d = g_{m1,2}\left(\frac{1}{g_{m3,4}}\right)$$

since $g_m = \sqrt{2\,\mu\, C_{OX}\,(W/L)\,I_D}$,

$$A_d = \frac{g_{m1,2}}{g_{m3,4}} = \frac{\sqrt{2\,\mu_n\, C_{OX}\,(W/L)_{1,2}\,I_D}}{\sqrt{2\,\mu_p\, C_{OX}\,(W/L)_{3,4}\,I_D}}$$

$$= \sqrt{\frac{\mu_n}{\mu_P}\cdot\frac{(W/L)_{1,2}}{(W/L)_{3,4}}}$$

(c) If $\mu_n = 4\mu_p$ and $L_1 = L_2 = L_3 = L_4 = L$,

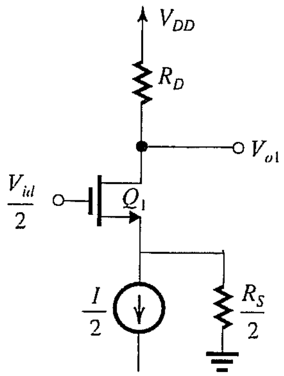
$$A_d = 10 \text{ V/V} = \sqrt{\frac{4\mu_n}{\mu_p}\cdot\frac{(W/L)_{1,2}}{(W/L)_{3,4}}}$$

$$\frac{10}{2} = \sqrt{\left(\frac{W_{1,2}}{W_{3,4}}\right)}$$

$$\left(\frac{W_{1,2}}{W_{3,4}}\right) = 25$$

8.21

HALF-CIRCUIT



small-signal analysis

$$V_{gs} = \frac{V_{id}}{2} - g_m V_{gs}\frac{R_s}{2}$$

$$V_{gs} = \frac{V_{id}/2}{1 + g_m\frac{R_s}{2}}$$

$$V_{O1} = -g_m V_{gs} R_D = -g_m\left[\frac{V_{id}/2}{1 + g_m R_s/2}\right]R_D$$

$$A_d = \frac{V_{Od}}{V_{id}} = \frac{g_m R_D}{1 + g_m R_s/2}$$

when $R_S = 0$ $A_d = g_m R_D$ (agrees with Eqtn. 8.35)

when $R_S = \frac{2}{g_m}$ the differential gain is reduced by half

8.22

(a) $V_{G1} = V_{G2} = OV$

$V_{S1} = V_{S2}$ assuming matching components

$$V_{S1} = V_{G1} - V_{GS1} = OV - (V_t + V_{OV})$$
$$= -(V_t + V_{OV})$$

(b) zero current flows through Q_3

$$V_{OV3} = V_C - V_{S1} - V_t = V_C - (-(V_t + V_{OV})) - V_t$$
$$= V_C + V_t$$
$$= V_C + V_{OV}$$

(c) $V_{G1} = -V_{G2} = V_{id}/2$

V_{S1} is now more negative than in (a) and V_{S2} is now less negative than in (a) so there is a voltage across Q_3. If this voltage is small and if V_C is such that $V_{GS3} > V_t$ then Q_3 will operate in triode.

$$r_{Ds3} = \left[k_n' \frac{W}{L} V_{ov3}\right]^{-1}$$

$$g_{m1} = g_{m2} = \frac{1/2\, k_n' \frac{W}{L} V_{ov}^2}{V_{ov}} = 1/2\, k_n' \frac{W}{L} V_{ov}$$

$$\text{so } r_{Ds3} = \left[g_{m1}\frac{V_{ov3}}{V_{ov}}\right]^{-1} = \frac{V_{ov}}{V_{ov3}\, g_{m1}}$$

(d) $r_{DS3} = \frac{V_{OV}}{V_{OV3}}\cdot\frac{1}{g_{m1}}$

$(i) R_S = \frac{1}{g_{m1}}$ $\therefore V_{OV3} = V_{OV}$

From (b) $V_{OV3} = V_C + V_{OV}$ so $V_C = 0\text{V}$

(ii) $R_s = \frac{1}{2\,g_{m1}}$ $\therefore V_{Ov3} = 2\,V_{OV}$

so $V_C = V_{OV}$

8.23

(a) $V_{G1} = V_{G2} = 0\text{V}$

$V_{S1} = V_{S2} = -(V_t + V_{OV})$

Zero current flows through Q_3 and Q_4

Q_3 and Q_4 have the same overdrive voltage as Q_1 and Q_2

$$r_{DS3} = r_{DS4} = \left[k_n'\left(\frac{W}{L}\right)_{3,4} V_{OV3,4}\right]^{-1}$$

	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	
μC_{ox}	30	30	30	90	90	30	90	μA/V²
I_D	50	50	100	50	50	100	100	μA
V_{OV}	–0.25	–0.25	–0.3	0.3	0.3	–0.3	0.3	V
$\frac{W}{L}$	53.3	53.3	74.1	12.3	12.3	74.1	24.7	
V_{GS}	–0.95	–0.95	–1	1	1	–1	1	V

Since $g_m = \dfrac{|I_D|}{|V_{OV}|/2}$,

$$|V_{OV1}| = |V_{OV2}| = |V_{OV4}| = |V_{OV5}| = \frac{2\,I_D}{g_m}$$

$$= \frac{2(50\ \mu A)}{400\ \mu A/V} = 0.25\ V$$

so,

$$V_{GS1} = V_{GS2} = V_{OV} + V_{tp} = -0.25 - 0.7$$

$$= -0.95\ V$$

For $\left(\frac{W}{L}\right)$ ratios

$$I_D = \frac{1}{2}\,\mu\,C_{ox}\left(\frac{W}{L}\right)(V_{OV})^2$$

So that

$$\frac{W}{L} = \frac{2\,I_D}{\mu\,C_{ox}V_{OV}^2}$$

For Q_7,

$$\left(\frac{W}{L}\right)_7 = \frac{2(100\ \mu A)}{90\ \mu A/V^2(0.3\ V)^2} = 24.7$$

For Q_4 and Q_5,

$$\left(\frac{W}{L}\right)_4 = \left(\frac{W}{L}\right)_5 = \frac{2(50\ \mu A)}{90\ \mu A/V^2(0.3)^2} = 12.3$$

For Q_1 and Q_2,

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{2(50\ \mu A)}{30\ \mu A/V^2(0.25)^2} = 53.3$$

For Q_6 and Q_3,

$$\left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_3 = \frac{2(100\ \mu A)}{30\ \mu A/V^2(0.3\ V)^2} = 74.1$$

In summary, the results are as follows:

8.25

(a) $I_{D1} = \frac{1}{2}k_n'\frac{W}{L}(V_{GS1} - V_t)^2$

$$I_{D2} = \frac{1}{2}k_n'\left(2 \times \frac{W}{L}\right)(V_{GS2} - V_t)^2$$

Since $V_{GS} - V_t$ is equal for both transistors :

$$\Rightarrow \frac{I_{D1}}{I_{D2}} = \frac{1}{2};\ I_{D2} = 2I_{D1}$$

but $I = I_{D1} + I_{D2} = 3I_{D1}$

$$I_{D1} = I/3$$

$$I_{D2} = 2I/3$$

(b) $V_{OV} = V_{GS} - V_t$

$$V_{OV1} = V_{OV2} = V_{OV}$$

For $Q1$: $\frac{I}{3} = \frac{1}{2}k_n'\left(\frac{W}{L}\right)V_{OV}^2$

$$\Rightarrow V_{OV} = \sqrt{\frac{2}{3}\frac{I}{k_n' W/L}}$$

(c) $g_m = \dfrac{2I_D}{V_{OV}} \rightarrow g_{m1} = \dfrac{2I}{3V_{OV}}$

$$g_{m2} = \frac{4}{3}\frac{I}{V_{OV}}$$

$$v_{O1} = -g_{m1} \times \frac{v_{id}}{2} \cdot R_D$$

$$= -\frac{2}{3}\frac{I}{V_{OV}} \cdot R_D \cdot v_{id}$$

$$v_{O2} = +\,g_{m2} \times \frac{v_{id}}{2} \cdot R_D$$

$$= \frac{4}{3}\frac{I}{V_{OV}} \cdot R_D \cdot v_{id}$$

$$\Rightarrow \frac{v_{O2} - v_{O1}}{v_{id}} = \left(\frac{4}{3} + \frac{2}{3}\right)\frac{I}{V_{OV}} \cdot R_D$$

$$= 2 \times \frac{I}{V_{OV}} \cdot R_D$$

8.26

By equation 8.38 $A_d = g_{m1}(R_{on} \| R_{op})$

$$= g_{m1}[(g_{m3}r_{O3})r_{O1} \| (g_{m5}r_{O5})r_{O7}]$$

If all transistors have the same channel length and the same $|V_{OV}|$ and $|V_A|$ Since $g_m = \dfrac{2I_D}{V_{OV}}$ and

$r_O = \dfrac{V_A}{I_D}$ and with g_m and r_o the same for all devices,

$$A_d = \frac{2I_D}{V_{OV}}\left(\left(\frac{2I_D}{V_{OV}} \cdot \frac{V_A}{I_D}\right)\frac{V_A}{I_D}\right) \| \left(\left(\frac{2I_D}{V_{OV}} \cdot \frac{V_A}{I_D}\right)\frac{V_A}{I_D}\right)$$

$$= \frac{2I_D}{V_{OV}}\left[\frac{2\,V_A^2}{V_{OV}\,I_D} \| \frac{2\,V_A^2}{V_{OV}\,I_D}\right]$$

$= \left(\frac{2I_D}{V_{OV}} \cdot \frac{V_A^2}{V_{OV} I_D}\right)$

$= \frac{2 V_A^2}{V_{OV}^2} = 2\left(\frac{|V_A|}{|V_{OV}|}\right)^2$

For $A_d = 1000$ V / V and $|V_{OV}| = 0.2$ V

$1000 = 2\frac{|V_A|^2}{|V_{OV}|^2}$

$V_A = \sqrt{500} \cdot 0,2\text{ V} = 4.47\text{ V}$

If $|V_A'| = 10\text{ V}/\mu\text{A}$

$\text{L} = \frac{4.47\text{ V}}{10\text{ V}/\mu\text{M}} = 0.447\ \mu\text{m}$

For high g_m the bias current should be high, but with ± 0.9 V Supplies the bias current must not exceed $\frac{1\text{ mW}}{1.8\text{ V}} = 0.556$ mA to keep power dissipation at 1 mW

8.27

$I = 0.2\text{ mA}\ R_{SS} = 100\text{ k}\Omega\ R_D = 10\text{ k}\Omega$

$k_n'\frac{W}{L} = 3\frac{\text{mA}}{V^2}$ 1% mismatch in drain resistances

$V_{OV} = \sqrt{I/\text{k}_n'\frac{W}{L}} = \sqrt{\frac{0.2\text{ mA}}{3\text{ mA}/\text{V}^2}} = 0.258\text{ V}$

$g_m = \frac{I}{V_{OV}} = \frac{0.2\text{ mA}}{0.258\text{ V}} = 0.755\text{ mA}/\text{V}$

Equation 8.35

$|A_d| = g_m R_D = 0.775\frac{\text{mA}}{\text{V}} \cdot 10\text{ k}\Omega = 7.75\text{ V}/\text{V}$

Equation. 8.49a $|A_{CM}| = \frac{R_D}{2R_{SS}} \cdot \frac{\Delta R_D}{R_D}$

$= \frac{10\text{ k}\Omega}{2.100\text{ k}\Omega} \cdot 1\,\%$

$= 0.0005 = 0.5\text{ mV}/\text{V}$

Equation 8.50a

$CMRR = \frac{|A_d|}{|A_{CM}|} = \frac{7.75}{0.0005} = 15500 = 83.8\text{ dB}$

8.28

$\text{k}_p'\frac{W}{L} = 4\text{ mA}/\text{V}^2$ Currrent source resistance 30 kΩ

$|V_{OV}| = \sqrt{I/\text{k}_p'\frac{W}{L}} = \sqrt{0.5\text{ mA}/4\text{ mA}/\text{V}^2}$

$= 0.353\text{ V}$

$g_m = I/|V_{OV}| = \frac{0.5\text{ mA}}{0.418\text{ V}} = 1.42\text{ mA}/\text{V}$

$|A_d| = g_m R_D = (1.42)(4\text{ k}) = 5.68\text{ V}/\text{V}$

$|A_{CM}| = \frac{R_D}{2R_{SS}} \cdot \frac{\Delta R_D}{R_D} = \frac{4}{2 \times 30} \cdot 2\,\%$

$= 1.33\text{ mV}/\text{V}$

$CMRR = \frac{|\text{A}_d|}{|A_{CM}|} = \frac{5.68}{1.33 \times 10^{-3}} = 4.27 \times 10^3$

$= 72.6\text{ dB}$

8.29

(a) $I_{D1} = I_{D2} = \frac{1\text{ mA}}{2} = 0.5\text{ mA}$

$= \frac{1}{2}K_n'\frac{W}{L}V_{OV}^2 = \frac{1}{2}(2.5\text{ mA}/\text{V}^2)V_{OV}^2$

$V_{OV} = \left[\frac{2 \times 0.5\text{ mA}}{2.5\text{ mA}/\text{V}^2}\right]^{1/2} = \sqrt{0.4\text{V}^2} = 0.632\text{ V}$

$V_{GS} = V_t + V_{OV} = 0.7\text{ V} + 0.632\text{ V} = 1.332\text{ V}$

$V_S = (1\text{ mA})(1\text{ k}\Omega) = 1\text{ V}$

$V_{cn} = V_S + V_{GS} = 1\text{ V} + 1.332\text{ V} = 2.332\text{ V}$

(b) $A_d = g_m R_D = \frac{I}{V_{OV}}R_D$

$R_D = A_d \cdot \frac{V_{OV}}{I} = 8\text{ V}/\text{V} \cdot \frac{0.632\text{V}}{1\text{ mA}} = 5.06\text{ k}\Omega$

(c) $V_D = V_{DD} - I_D R_D$

$= 5\text{V} - (0.5\text{ mA})(5.06\text{ k}\Omega)$

$= 2.47\text{ V}$

(d) From Equation. 8.43

$\frac{\Delta V_{D1}}{\Delta V_{CM}} = -\frac{R_D}{\frac{1}{g_m} + 2R_{SS}} = -\frac{-5.06\text{ k}\Omega}{\frac{0.632\text{ V}}{1\text{ mA}} + 2(1\text{ k}\Omega)}$

$= -1.92\text{V}/\text{V}$

(e) Triode when $V_G - V_D = V_t$

$(V_{CM} + \Delta V_{CM}) - (V_D - A_{CM} \cdot \Delta V_{CM}) = V_t$

$(2.332\text{v} + \Delta V_{CM}) - (2.47\text{ V} - 1.92 \cdot \Delta V_{CM})$

$= 0.7\text{ V}$

$2.92\ \Delta V_{CM} = 0.7\text{V} - 2.332\text{ V} + 2.47\text{V}$

$= 0.838\text{V}$

$\Delta V_{Cm} = 0.838\text{ V}/2.92 = 0.287\text{ V}$

8.30

(a) $R_{D1} = R_D + \frac{\Delta R_D}{2}\ R_{D2} = R_D - \frac{\Delta R_D}{2}$

$g_{m1} = g_m + \frac{\Delta g_m}{2}\ g_{m2} = g_m - \frac{\Delta g_m}{2}$

$i_{d1} = \frac{g_{m1} V_{icm}}{g_m R_{SS}}\ i_{d2} = \frac{g_{m2} V_{icm}}{2g_m R_{SS}}$

$i_{d1} - i_{d2} = (g_{m1} - g_{m2})\frac{V_{icm}}{2g_m R_{SS}}$

8.70

The bias current I will split between the two differential transistors according to their base-emitter areas. So, the larger device will carry $\frac{2I}{3}$ Amperes,

while the second transistor will carry $\frac{I}{3}$ A

Assuming, for example, that Q_1 has the larger area, $r_{e1} = \frac{V_T}{2I/3}$ and $r_{e2} = \frac{V_T}{I/3}$

Normally, we could apply the common-mode half circuit symmetry. But here, the amplifier is not symmetrical.

If $r_{e1} = \frac{3V_T}{2I}$ and $r_{e2} = \frac{3V_T}{I}$,

$i_{c1} \neq i_{c2}$

so, $V_{od} = -i_{c2} R_c - (-i_{c1} R_c) = (i_{c1} - i_{c2}) R_c$

with $\alpha \simeq 1$

$$i_{c1} + i_{c2} \simeq i_{e1} + i_{e2} = \frac{V_{Icm}}{R_{EE}} \text{ (since } R_{EE} >> r_e)$$

$$A_{cm} = \left|\frac{V_{od}}{V_{Icm}}\right| = \frac{\frac{1}{3}R_C}{R_{EE}} = \frac{12\text{ K}}{3(500\text{ K})} = 0.008\text{ V/V}$$

8.71

For $I = 160\mu\text{A}$,

$$I_D = \frac{I}{2} = \frac{160\ \mu\text{A}}{2} = 80\ \mu\text{A}$$

$$g_m = \sqrt{2k_n'\frac{W}{L}I_D} = \sqrt{2(4\text{ mA/V}^2)(80\ \mu\text{A})}$$

$= 0.8$ mA/V

$R_D = 10\text{ k}\Omega$, so that

$A_d = g_m R_D = (0.8\text{ mA/V})(10\text{ k}) = 8$ V/V

using Eq. (8.108),

$$V_{OS} = \left(\frac{V_{ov}}{2}\right)\left(\frac{\Delta R_D}{R_D}\right) \text{ where } \frac{\Delta R_D}{R_D} = 0.02$$

(worst-case)

since $g_m = \frac{I_D}{V_{ov}/2}$,

$$\frac{V_{ov}}{2} = \frac{I_D}{g_m} = \frac{80\ \mu\text{A}}{0.8\text{ mA/V}} = 0.1\text{ V}$$

Therefore, $V_{os} = (0.1\text{ V})(0.02) = 2$ mV

For $I = 360\ \mu\text{A}$, $I_D = \frac{360\ \mu\text{A}}{2} = 180\ \mu\text{A}$

$$g_m = \sqrt{2k_n'\frac{W}{L}I_D} = \sqrt{2(4\text{ mA/V}^2)(0.18\text{ mA})}$$

$= 1.2$ mA/V

$A_d = g_m R_D = (1.2\text{ mA/V})(10\text{ k}) = 12$ V/V

$$\frac{V_{ov}}{2} = \frac{I_D}{g_m} = \frac{180\ \mu\text{A}}{1.2\text{ mA/V}} = 0.15\text{ V}$$

so that

$$V_{os} = \left(\frac{V_{os}}{2}\right)\left(\frac{\Delta R_D}{R_D}\right) = (0.15\text{ V})(0.02) = 3\text{ mV}$$

So both A_d and V_{os} increase at the same ratio since both are proportional to $\sqrt{I_D}$

8.72

Worst-case $\Delta V_T = 10$ mV

Worst-case $\frac{\Delta R_D}{R_D} = 0.04$

Worst-case $\frac{\Delta(W/L)}{(W/L)} = 0.04$

From Eq. (8.108),

$$V_{OS1}(d_{ve}\text{ to }\Delta R_D) = \frac{V_{OV}}{2}\cdot\frac{\Delta R_D}{R_D} = \frac{0.2\text{ V}}{2}\cdot(0.04)$$

$= 4$ mV

From Eq. (8.113),

$$V_{OS2}\left(d_{ve}\text{ to }\Delta\left(\frac{W}{L}\right)\right) = \frac{V_{OV}}{2}\cdot\frac{\Delta(W/L)}{(W/L)}$$

$$= \frac{0.2\text{ V}}{2}\cdot(0.04) = 4\text{ mV}$$

From Eq. (8.116),

$V_{OS3}(d_{ve}\text{ to }\Delta V_t) = \Delta V_t = 10$ mV

The absolute worst-case total offset would be $4 + 4 + 10 = 18$ mV However, since these offset sources are not correlated, a realistic observation might be [from Eq. (8.117)]

$$V_{OS} = \sqrt{V_{OS1}^2 + V_{OS2}^2 + V_{OS3}^2}$$

$$= \sqrt{(4)^2 + (4)^2 + (10)^2} = 11.5\text{ mV}$$

The major contribution is from the variation in V_T. If we attempt to compensate for V_{OS} by Changing R_D,

We have, $11.5\text{ mV} = \frac{V_{OV}}{2}\cdot\frac{\Delta R_D}{R_D}$ or

$$\frac{\Delta R_D}{R_D} = \frac{11.5\text{ mV}}{0.2\text{ V}/2} = 0.115 \text{ or } 11\%$$

If V_{OS3} is reduced by a factor of 10,

$$V_{OS} = \sqrt{(4)^2 + (4)^2 + (1)^2} = 5.74\text{ mV}$$

So that

$$\frac{\Delta R_D}{R_D} = \frac{5.74\text{ mV}}{0.2\text{ V}/2} = 5.74\,\%$$

8.73

$$I_D = \frac{I}{2} = \frac{100\ \mu\text{A}}{2} = 50\ \mu\text{A}$$

$$I_D = \frac{1}{2}K_n'\left(\frac{W}{L}\right)V_{OV}^2 \text{ or}$$

$$V_{OV} = \sqrt{\frac{2I_D}{K_n'(W/L)}} = \sqrt{\frac{2(50\ \mu\text{A})}{250\ \mu\text{A/V}^2(10)}}$$

$= 0.2\text{ V}$

From Eq.(8.108)

$$V_{OS1}(due\text{ to }\Delta R_D) = \frac{V_{OV}}{2} \cdot \frac{\Delta R_D}{R_D} = \frac{0.2\text{ V}}{2} \cdot (0.05)$$

$= 5\text{ mV}$

From Eq. (8.113),

$$V_{OS2}\left(due\text{ to }\Delta\frac{W/L}{W/L}\right) = \frac{V_{OV}}{2} \cdot \frac{\Delta(W/L)}{(W/L)}$$

$$= \frac{0.2\text{ V}}{2} \cdot (0.05) = 5\text{ mV}$$

From Eq. (8.116),

$V_{OS3}(\text{due to }\Delta V_t) = \Delta V_t = 5\text{ mV}$

The worst-case offset is

$$V_{OS} = V_{OS1} + V_{OS2} + V_{OS3} = 5 + 5 + 5$$

$= 15\text{ mV}$

The root-sum-square value from Eq. (8.117) is

$$V_{OS} = \sqrt{V_{OS1}^2 + V_{OS2}^2 + V_{OS3}^2}$$

$$= \sqrt{(5)^2 + (5)^2 + (5)^2} = 8.66\text{ mV}$$

8.74

The output offset voltage is $\Delta V_C = \Delta R_C \cdot \frac{I}{2}$

$$A_d = g_m R_C = \frac{I/2}{V_T} \cdot R_C = \frac{IR_C}{2V_T}$$

$$|V_{OS}| = \frac{|\Delta V_C|}{A_d} = \frac{\frac{\Delta R_C I}{2}}{\frac{I R_C}{2 V_T}}$$

$$= V_T \cdot \left(\frac{\Delta R_C}{R_C}\right) = 25\text{ mV}(0.08)$$

$|V_{OS}| = 2\text{ mV}$

8.75

From Eq. (8.126),

$$|V_{OS}| = V_T\left(\frac{\Delta I_S}{I_S}\right)$$

$$|V_{OS}| = 25\text{ mV}(0.10) = 2.5\text{ mV}$$

8.76

$$\Delta v_{EE} = \Delta R_C \frac{I}{2}$$

$$A_d = \frac{R_C}{r_e + R_e} = \frac{R_C}{\frac{2V_T}{I} + R_E} = \frac{I R_C}{2 V_T + I R_E}$$

$$V_{OS} = \frac{\Delta v_C}{A_d} = \frac{\Delta R_C}{R_C}\left(V_T + \frac{I R_E}{2}\right)$$

8.77

$$\Delta v_C = \alpha_1 \frac{I}{2} R_C - \alpha_2 \frac{I}{2} R_C$$

$$= \frac{I}{2} R_C(\alpha_1 - \alpha_2)$$

$$= \frac{I}{2} R_C\left(\frac{\beta_1}{\beta_1 + 1} - \frac{\beta_2}{\beta_2 + 1}\right)$$

For $\beta_1, \beta_2 >> 1$

$$\Delta v_C = \frac{I}{2} R_C \cdot \frac{\beta_1 - \beta_2}{\beta_1 \cdot \beta_2}$$

$$= \frac{I}{2} R_C\left(\frac{1}{\beta_2} - \frac{1}{\beta_1}\right)$$

$$A_d = \frac{R_C}{r_e} = \frac{I R_C}{2 V_T}$$

$$V_{OS} = \frac{\Delta v_C}{A_d} = V_T\left(\frac{1}{\beta_2} - \frac{1}{\beta_1}\right) \text{ Q.E.D.}$$

For $\beta_1 = 100$ and $\beta_2 = 200$

$$V_{OS} = 25\left(\frac{1}{200} - \frac{1}{100}\right)$$

$= -125\ \mu\text{V}$

8.78

CASE 1: BJT Diff. Amp.

From Eq. (8.121)

$$|V_{OS}| = V_T\left(\frac{\Delta R_C}{R_C}\right) = 25\text{ mV }(0.04) = 1\text{ mV}$$

CASE 2: MOSFET Diff. Amp.

From Eq. (8.108),

$$V_{OS} = \left(\frac{V_{OV}}{2}\right)\left(\frac{\Delta R_D}{R_D}\right) = \frac{300\text{ mV}}{2}\ (0.04) = 6\text{ mV}$$

If the MOSFET widths, are increased by a factor of 4, and since I_D must remain constant, we see that since

$$I_D = \frac{1}{2} K_n'\left(\frac{W}{L}\right) V_{OV}^2,$$

The new $V_{OV} = \sqrt{\dfrac{2I_D}{(4)K_n'\left(\frac{W}{L}\right)}}$ which is $\sqrt{\frac{1}{4}}$ or $\frac{1}{2}$

of its orginal value.

So, the new offset voltage is

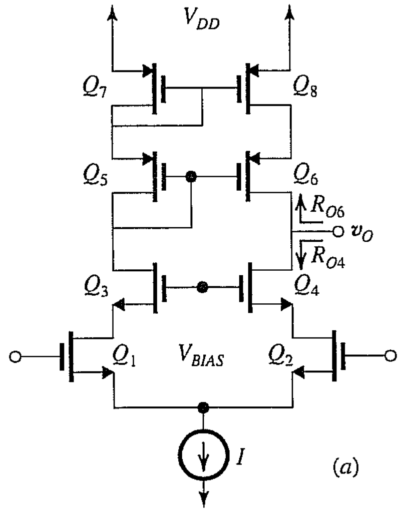
$$V_{OS} = \left(\frac{150\text{ mV}}{2}\right)(0.04) = 3\text{ mV}$$

8.79

Since the two transistors are matched except for their V_A value, we can express the collector currents when the input terminals are grounded as,

$$I_{C1} = I_C\left(1 + \frac{V_{CE}}{V_{A1}}\right)$$

8.89



(*a*)

(b) $R_{O4} = (g_{m4}r_{O4})r_{O2}$
$= g_m r_o^2$
$R_{O6} = (g_{m6}\, r_{O6})\, r_{O8}$
$= g_m r_o^2$
$A_d = g_m(R_{O4} \| R_{O6})$
$= g_m \cdot \frac{1}{2} gm^2 r_o^2$

$g_m = \frac{2I_D}{V_{OV}} \qquad r_O = \frac{V_A}{I_D}$

thus, $g_m r_O = 2V_A / V_{OV}$

$\Rightarrow A_d = 2(V_A / V_{OV})^2$

Q.E.D.
For $V_{OV} = 0.25$ V & $V_A = 20$ V
$A_d = 2\,(20 / 0.25)^2 = 12800$ V/V

8.90

Referring to Fig P8.90,

$$i_1 = \frac{V_O}{r_O} = \frac{\frac{1}{2}(g_m r_O)\, v_{id}}{r_O} = \frac{1}{2} g_m v_{id}$$

$$i_2 = g_{m4}\, v_{sg4} = \frac{g_m\, v_{id}}{4}$$

$$i_3 = i_1 - i_2 = \frac{g_m\, v_{id}}{2} - g_m \frac{v_{id}}{4} = \frac{g_m\, v_{id}}{4}$$

$$i_4 = -g_{m2}\, v_{gs2} = -g_m\left[-\frac{v_{id}}{2} - \frac{v_{id}}{4}\right]$$

$$= \frac{3}{4} g_m\, v_{id}$$

$$i_5 = i_4 = \frac{3}{4} g_m\, v_{id}$$

$$i_6 = i_4 - i_3 = \frac{3}{4} g_m\, v_{id} - \frac{1}{4} g_m v_{id} = \frac{1}{2} g_m v_{id}$$

However, if we use KVL,

$$i_6 = \frac{v_o - v_s}{r_o} = \frac{\frac{1}{2} g_m\, r_o\, v_{id} - \frac{V_{id}}{4}}{r_o}$$

$$= \frac{1}{2} g_m\, V_{id} - \frac{V_{id}}{4r_o} \quad \text{inconsistent}$$

$$i_7 = i_5 - i_6 = \frac{3}{4} g_m\, V_{id} - \frac{g_m\, V_{id}}{2} = \frac{g_m\, V_{id}}{4}$$

(which is the same as i_3)

$$i_8 = g_m v_{gs1} = g_m\left(\frac{V_{id}}{2} - \frac{V_{id}}{4}\right) = \frac{1}{4} g_m v_{id}$$

$$i_9 = i_8 = \frac{1}{4} gm\; V_{id}$$

$$i_{10} = i_8 - i_7 \doteq \frac{g_m\, V_{id}}{4} - \frac{g_m\, V_{id}}{4} = 0$$

$i_{11} + i_{10} = i_9$ or

$$i_{11} = i_9 - i_{10} = i_9 = \frac{g_m\, V_{id}}{4}$$

(which is the same as i_7)

$$i_{12} = g_m v_{gs3} = \frac{1}{4} g_m\, V_{id}$$

$$i_{13} = i_{11} - i_{12} = \frac{1}{4} g_m\, V_{id} - \frac{1}{4} g_m\, V_{id} = 0$$

Note, through, that this is inconsistent with KVL. If $i_{13} = 0$, $V_{D3} = 0$, but $V_{D3} = V_{G3} = -V_{id}/4$.

If $i_{10} = 0$, $V_{D1} = \frac{V_{id}}{4}$, but this conflicts with V_{D3} being $-\frac{V_{id}}{4}$.

It appears that the approximations for V_{gs} and v_s prevent a clean solution. If these were more exact, all current and voltage relationships should be consistent.

8.91

Assuming a configuration similar to Fig. 8.32(a),

$$I_{D1} = I_{D2} = I_{D3} = I_{D4} = \frac{I}{2} = \frac{100\ \mu\text{A}}{2} = 50\ \mu\text{A}$$

$$g_{m1} = g_{m2} = \frac{I_D}{V_{ov}/2} = \frac{50\ \mu\text{A}}{0.2\ \text{V}/2} = 0.5\ \text{mA}/\text{V}$$

$$G_m = g_{m1} = 0.5\ \text{mA}/\text{V}$$

$$r_{o1} = r_{o2} = \frac{V_{An}}{I_D} = \frac{20\ \text{V}}{0.05\ \text{mA}} = 400\ \text{k}\Omega$$

$$r_{o3} = r_{o4} = \frac{|V_{Ap}|}{I_D} = \frac{12\ \text{V}}{0.05\ \text{mA}} = 240\ \text{k}\Omega$$

From Eq. (8.140),

$$R_o = r_{o2} \| r_{o4} = 400\ \text{k} \| 240\ \text{k} = 150\ \text{k}\Omega$$

$$A_d = G_m R_o = (0.5\ \text{mA}/\text{V})(150\ \text{k}) = 75\ \text{V}/\text{V}$$

Gain will be reduced by a factor of 2 if
$R_L = R_o = 150\ \text{k}\Omega$

8.110

$$I_{D7} = \frac{W_7}{W_8} I_{REF} = \frac{50}{40} \times 90\ \mu A$$
$$= 112.5\ \mu A$$

Output offset current $= I_{D7} - I_{D6}$

$$= 112.5 - 90 = 22.5\ \mu A$$
$$\Rightarrow V_o = 22.5\ \mu\ (r_{o6} \| r_{o7})$$
$$r_{o7} = \frac{10}{112.5\ \mu} = 88.9\ k\Omega$$
$$\Rightarrow V_o = 22.5\ \mu\ (111\ k \| 88.9\ k)$$
$$= 1.11\ V$$
$$V_{os} = \frac{V_o}{A_o} = \frac{1.11 V}{1109} = 1\ mV$$

8.111

Offset current $= I_{D2} - I_{D4}$

$= I_{D3} - I_{D4}$

$$I_{D3} = \frac{K}{2}(V_{GS} - V_t)^2$$
$$I_{D4} = \frac{K}{2}(V_{GS} - (V_t + \Delta V_t))^2$$
$$I_o = I_{D3} - I_{D4}$$
$$= \frac{K}{2}[(V_{GS} - V_t - V_{GS} + V_t + \Delta V_t) \times$$
$$(V_{GS} - V_t + V_{GS} - V_t - \Delta V_t)]$$
$$= \Delta V_t \cdot \frac{K}{2}(2V_{GS} - 2V_t - \Delta V_t)$$
$$\simeq K(V_{GS} - V_t) \cdot \Delta V_t$$
$$I_o = g_{m3}\, \Delta V_t$$

Recall $I_o = G_{m1} \cdot V_{os}$

and $G_{m1} = g_{m1}$

$$\Rightarrow V_{os} = \frac{g_{m3}}{g_{m1}} \cdot \Delta V_t$$

For $\Delta V_t = 2\ mV$

$$V_{os} = \frac{0.3\ m}{0.3\ m} \times 2\ m = 2\ mV$$

8.112

(a) Referring to Fig. P8.112,

$$I_{E1} = I_{E2} = \frac{I}{2} = \frac{0.2\ mA}{2} = 0.1\ mA$$
$$I_{E3} \simeq I_{E4} = \frac{I}{2} = 0.1\ mA$$
$$I_{E5} = 0.5\ mA$$

since the output voltage is held at 0 V,

$I_{E6} = 1\ mA$

(b) considering the first stage,

$$G_{m1} = g_{m1} = \frac{|I_{C1}|}{V_T} = \frac{0.1\ mA}{25\ mV} = 4\ mA/V$$

since all $r_{os} = \infty$, the load of the differential stage is just the input of Q_5.

$$r_{e5} = \frac{V_T}{|I_{E5}|} = \frac{25\ mV}{0.5\ mA} = 50\ \Omega$$
$$R_{o1} = (\beta + 1)r_{e5} = (101)(50) = 5.05\ k\Omega$$
$$A_1 = G_{m1}R_{o1} = (4\ mA/V)(5.05\ k) = 20.2\ V/V$$

For the common-emitter stage (Q_5),

$$g_{m5} = \frac{I_{C5}}{V_T} = \frac{0.5\ mA}{25\ mA} = 20\ mA/V$$

The load is essentially

$$(R_L + r_{e6})(\beta + 1) \simeq R_L(\beta + 1)$$
$$A_5 = -g_{m5}(R_L)(\beta + 1)$$
$$= -20\ mA/V\ (10\ k)(101) = -20,200\ V/V$$

$A_6 \simeq 1$ so,

$$A_o = A_1 \cdot A_5 \cdot A_6 = (20.2)(-20,200)(1)$$
$$= 408,040\ V/V$$

or

$$-A_o(dB) = 20\log_{10}(408,040) = 112\ dB$$

8.113

$$I_B = 225\ \mu A$$
$$\mu_u C_{OX} = 180\ \mu A/V^2$$
$$\mu_p C_{OX} = 60\ \mu A/V^2$$

For Q_8 & Q_9 : W/L = 60/0.5

$$\Rightarrow |V_{ov}| = \sqrt{\frac{2I_D}{k_p'(W/L)}}$$
$$|V_{ov}|_{8,9} = \sqrt{\frac{2 \times 225\ \mu}{60\ \mu \times 120}} = 0.25\ V$$

then $g_{m8,9} = \dfrac{2I_D}{|V_{ov}|} = \dfrac{2 \times 225\ \mu}{0.25\ V}$

$= 1.8$ mA/V

Since g_m of Q_{10}, Q_{11} & Q_{13} are identical to g_m of Q_8 & Q_9 then $V_{ov13} = 0.25$ V

Thus for Q_{13}

$$(0.25)^2 = \frac{2 \times 225\ \mu}{180\ \mu \times (W/L)_{13}}$$
$$\rightarrow (W/L)_{13} = 40 \text{ i.e, } (20/0.5)$$

Since Q_{12} is 4 times as wide as Q_{13}, then

$$(W/L)_{12} = \frac{4 \times 20}{0.5} = 80/0.5$$
$$R_B = \frac{2}{\sqrt{2\, k_n'\, (W/L)_{12}\, I_B}} \cdot \left(\sqrt{\frac{(W/L)_{12}}{(W/L)_{13}}} - 1\right)$$
$$= \frac{2}{\sqrt{2 \times 180\ \mu \times \frac{80}{0.5} \times 225\ \mu}} \cdot \left(\frac{\sqrt{\frac{80/0.5}{20/0.5}} - 1}{\sqrt{4} - 1}\right)$$
$$\rightarrow R_B = 555.6\ \Omega$$

The voltage drop on R_B is :

$555.6 \times 225\mu = 0.125$ V.

To obtain the gate voltages:(assume $|V_{tn}| = |V_{tp}| =$ 0.7 V)