University of Toronto

Term Test 2

Date - Nov 18, 2013 (11:10am to 12pm)

Duration: 50 min

ECE331 — Analog Electronics
Lecturer - D. Johns

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

1. Equation sheet is on last page of test.
2. Unless otherwise stated, use transistor parameters on equation sheet.
3. Non-programmable calculator allowed; No other aids allowed
4. Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

Question	Mark
1	
2	
3	
4	
Total	

Last Name: SOLUTIONS

First Name: ____________________

Student #: ____________________

(max grade = 24)

[6] Question 1: Consider a first-order highpass circuit with input $v_s(t)$, output $v_o(t)$, time-constant $\tau = 1\mu s$ and a high freq gain of 3.

a) Give an expression for the transfer-function, $T(s)$ for this circuit.

$$T(s) = \frac{ks}{s + \omega_{3dB}} \qquad T(j\infty) = 3 = k \qquad k = 3$$

$$\omega_{3dB} = \frac{1}{\tau} = \frac{1}{1\mu s} = 1\,\text{M RAD/S}$$

$$T(s) = \frac{3s}{s + 1e6}$$

b) What is the f_{3dB} frequency for $T(s)$.

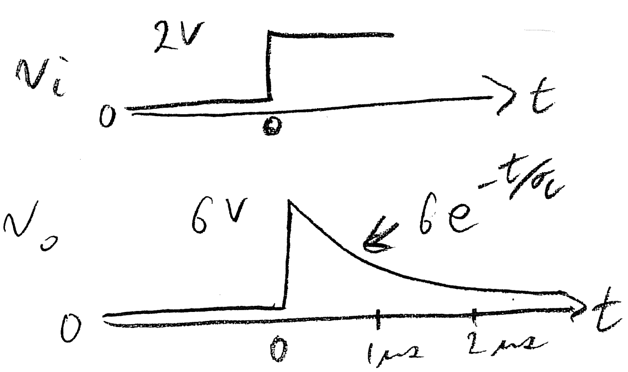
$$\omega_{3dB} = 1e6 = 2\pi f_{3dB}$$

$$f_{3dB} = \frac{1e6}{2\pi} = 159\,\text{kHz}$$

$f_{3dB} = 159\,\text{kHz}$

c) Sketch the step response for a step input of 2V. In terms of τ, find the time, t_{set}, it takes for $v_o(t)$ to settle to within 1% of its final value.

$t_{set} = 4.6\tau$

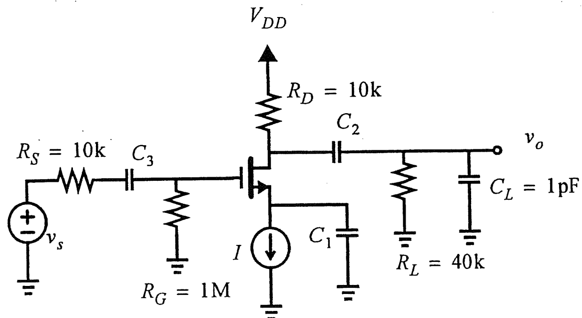


TO SETTLE TO 1%

$$e^{-t_{SET}/\tau} = 0.01 \Rightarrow t_{SET} = \tau(-\ln 0.01)$$

$$= 4.6\tau = 4.6\mu s$$

[6] Question 2: Consider the circuit shown below.



$C_{gs} = 1pF$
$C_{gd} = 0.2pF$
$C_{db} = 0.2pF$
$g_m = 0.5mA/V$ $\quad r_S = \frac{1}{g_m} = 2k$
ignore r_0
current source output impedance = 10k ← r_{oI}

Given that the low frequency poles should occur at 1Hz, 10Hz and 100Hz, and it is desired to minimize capacitor sizes, find the value of C_1 (explain your reasoning).

LOW FREQ POLES DUE TO

$C_1 \quad C_2 \quad C_3$

$C_1 = 6\mu F$

$$w_{p1} = \frac{1}{C_1 (r_S \| r_{oI})} = \frac{1}{C_1(1.67k)}$$

$$w_{p2} = \frac{1}{C_2 (R_D + R_L)} = \frac{1}{C_2(50k)}$$

$$w_{p3} = \frac{1}{C_3 (R_G)} = \frac{1}{C_3(1M)}$$

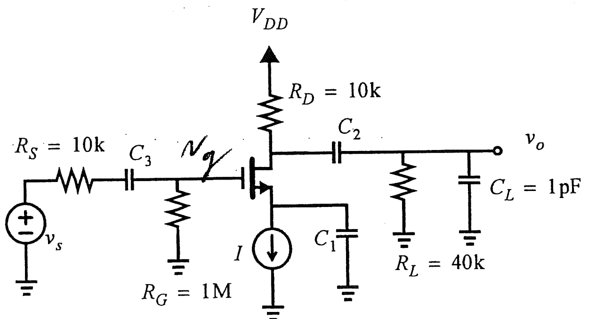
TO MINIMIZE CAP SIZES, LOWEST FREQ POLE SHOULD BE ASSOCIATED WITH LARGEST REQ WHERE $w_p = \frac{1}{C\,R_{EQ}}$, ETC

SO $w_{p1} = 100\,Hz \Rightarrow C_1 = \frac{1}{(100)(1.67k)} = 6\mu F$

$w_{p2} = 10\,HZ$

$w_{p3} = 1\,HZ$

[6] Question 3: Consider the circuit shown below.

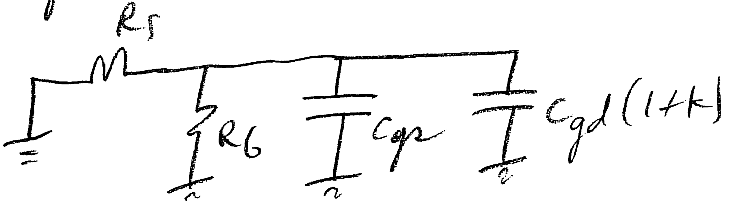


$C_{gs} = 1pF$
$C_{gd} = 0.2pF$
$C_{db} = 0.2pF$
$g_m = 0.5mA/V$ ← $r_s = \frac{1}{g_m} = 2k$
ignore r_0
current source output impedance = 10k

Estimate the 2 high frequency poles for this circuit (in Hz), f_{p1} and f_{p2}.

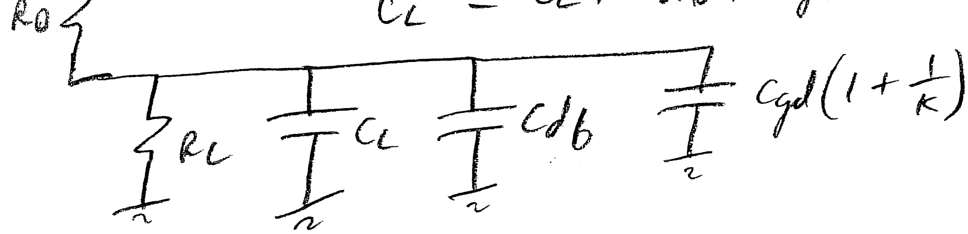
f_{p1} =	8MHz
f_{p2} =	13.7MHz

$$\frac{v_o}{v_g} = -\frac{R_D \| R_L}{r_s} = -4 \Rightarrow k = 4$$



$$f_{p1} = \frac{1}{2\pi\,(C_{gs} + C_{gd}(1+k))(R_S \| R_G)} \simeq 7.96\,MHz \simeq 8\,MHz$$

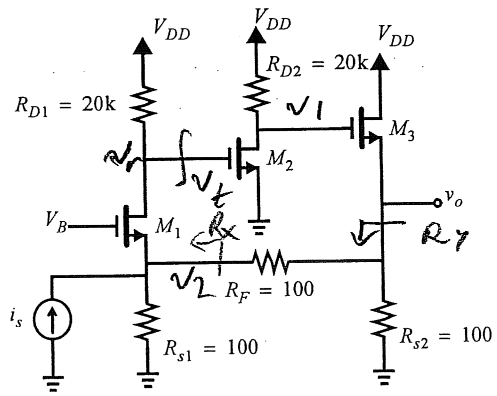
$$C_L' = C_L + C_{db} + C_{gd}\left(1 + \frac{1}{k}\right) = 1.45pF$$



$$f_{p2} = \frac{1}{2\pi\, C_L' (R_D \| R_L)} \cong 13.7\,MHz$$

OR 14.2 MHz IF $(1+\frac{1}{k}) \simeq 1$

[6] **Question 4:** Consider the feedback circuit below. Find the loop gain, L, asymptotic gain, A_∞, and feedthrough gain, d.



$g_{m1} = g_{m2} = g_{m3} = 2mA/V$ ← $r_s = \frac{1}{g_m} = 500$

ignore r_0

$L =$	83.3
$A_\infty =$	$-100\ \Omega$
$d =$	26

$$\frac{v_1}{v_t} = -\frac{R_{D2}}{r_{s2}} = -40\ V/V$$

$R_x = r_{s1} \| R_{s1} = 83.3\Omega$ $\qquad R_y = R_{s2} \| (R_F + R_x) = 64.7\Omega$

$$\frac{v_o}{v_1} = \frac{R_y}{R_y + r_{s3}} = 0.1146$$

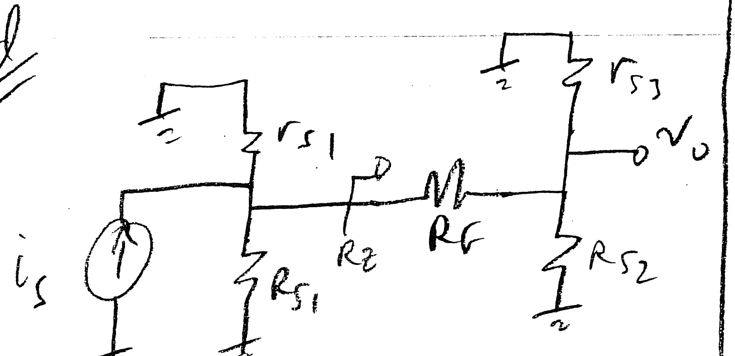
$$\frac{v_2}{v_o} = \frac{R_x}{R_x + R_F} = 0.4545 \qquad \frac{v_r}{v_2} = g_m R_{D1} = 40\ V/V$$

$$L \equiv -\frac{v_r}{v_t} = (40)(0.1146)(0.4545)(40) = 83.3$$

$A_\infty \Rightarrow v_r \to 0 \Rightarrow i_{s1} \to 0$ & $v_2 \to 0 \Rightarrow i_{R_{s1}} \to 0$

$v_o = -i_s R_F \qquad A_\infty \equiv \frac{v_o}{i_s} = -R_F = -100\ \Omega$

d



$d = \frac{v_o}{i_s}$

$$d = (r_{s1} \| R_{s1} \| R_z) \frac{(r_{s3} \| R_{s2})}{(r_{s3} \| R_{s2}) + R_F}$$

$$d = (57.3)(0.4545) = 26$$

$R_z = R_F + (R_{s2} \| r_{s3}) = 183.3\Omega$

Last Name: ____________

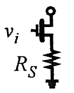
ECE331 Analog Electronics Equation Sheet

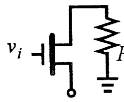
Constants: $k = 1.38 \times 10^{-23}\ \mathrm{JK}^{-1}$; $q = 1.602 \times 10^{-19}$ C ; $V_T = kT/q \approx 26\mathrm{mV}$ at 300 °K ;
$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m ; $k_{ox} = 3.9$; $C_{ox} = (k_{ox}\varepsilon_0)/t_{ox}$

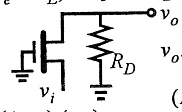
NMOS: $k_n = \mu_n C_{ox}(W/L)$; $V_{tn} > 0$; $v_{DS} \ge 0$; $v_{ov} = v_{GS} - V_{tn}$
(triode) $v_{DS} \le v_{ov}$ (or $v_D < v_G - V_{tn}$) ; $i_D = k_n((v_{ov})v_{DS} - (v_{DS}^2/2))$
(active) $v_{DS} \ge v_{ov}$; $i_D = 0.5k_n v_{ov}^2(1 + \lambda v_{DS})$; $g_m = k_n V_{ov} = 2I_D/V_{ov} = \sqrt{2k_n I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda'|I_D)$

PMOS: $k_p = \mu_p C_{ox}(W/L)$; $V_{tp} < 0$; $v_{SD} \ge 0$; $v_{ov} = v_{SG} - |V_{tp}|$
(triode) $v_{SD} \le v_{ov}$ (or $(v_D > v_G + |V_{tp}|)$) ; $i_D = k_p((v_{ov})v_{SD} - (v_{SD}^2/2))$
(active) $v_{DS} \ge v_{ov}$; $i_D = 0.5k_p v_{ov}^2(1 + |\lambda| v_{SD})$; $g_m = k_p V_{ov} = 2I_D/V_{ov} = \sqrt{2k_p I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda'|I_D)$

BJT: (active) $i_C = I_S e^{(v_{BE}/V_T)}(1 + (v_{CE}/V_A))$; $g_m = \alpha/r_e = I_C/V_T$; $r_e = V_T/I_E$; $r_\pi = \beta/g_m$; $r_o = |V_A|/I_C$
$i_C = \beta i_B$; $i_E = (\beta+1)i_B$; $\alpha = \beta/(\beta+1)$; $i_C = \alpha i_E$; $R_b = (\beta+1)(r_e + R_E)$; $R_e = (R_B + r_\pi)/(\beta+1)$

Cascode: 

$R_x \approx (1 + g_m R_S)r_o$
$i_{sc} \approx -(1/g_m + R_S)^{-1} v_i$


$R_x \approx 1/g_m + R_D/(g_m r_o)$
$v_{oc} \approx v_i$


$v_o/v_i \approx g_m(r_o \| R_D)$
(Approx due to $g_m r_o \gg 1$)

Diff Pair: $A_d = g_m R_D$; $A_{CM} = -(R_D/(2R_{SS}))((\Delta R_D)/R_D)$; $A_{CM} = -(R_D/(2R_{SS}))((\Delta g_m)/g_m)$
$V_{os} = \Delta V_t$; $V_{os} = (V_{ov}/2)((\Delta R_D)/R_D)$; $V_{os} = (V_{ov}/2)((\Delta(W/L))/(W/L))$

1st order: step response $y(t) = Y_\infty - (Y_\infty - Y_{0+})e^{-t/\tau}$ unity gain freq for $T(s) = \dfrac{A_M}{1 + s/\omega_{3dB}}$ $f_t \approx |A_M|\omega_{3dB}$ when $A_M \gg 1$

Freq: for real axis poles/zeros $T(s) = k_{dc}\dfrac{(1 + s/z_1)(1 + s/z_2)\ldots(1 + s/z_m)}{(1 + s/\omega_1)(1 + s/\omega_2)\ldots(1 + s/\omega_n)}$
OTC estimate $f_H = 1/(2\pi\sum\tau_i)$; dominant pole estimate $f_H = 1/(2\pi\tau_{max})$

Miller: $Z_1 = Z/(1-K)$; $Z_2 = Z/(1 - 1/K)$

Mos caps: $C_{gs} = (2/3)WLC_{ox} + WL_{ov}C_{ox}$; $C_{gd} = WL_{ov}C_{ox}$; $C_{db} = C_{db0}/(\sqrt{1 + V_{db}/V_0})$
$f_t = g_m/(2\pi(C_{gs} + C_{gd}))$ assuming $C_{gd} \ll C_{gs}$ $f_t = (3\mu V_{ov})/(4\pi L^2)$

Feedback: $A_f = A/(1 + A\beta)$; $x_i = (1/(1 + A\beta))x_s$; $dA_f/A_f = (1/(1 + A\beta))dA/A$; $\omega_{Hf} = \omega_H(1 + A\beta)$; $\omega_{Lf} = \omega_L/(1 + A\beta)$
Loop Gain $L \equiv -s_r/s_t$; $A_f = A_\infty(L/(1 + L)) + d/(1 + L)$; $Z_{port} = Z_{p0}((1 + L_S)/(1 + L_O))$
PM $= \angle L(j\omega_1) + 180$; GM $= -|L(j\omega_{180})|_{dB}$
Pole Splitting $\omega_{p1}' \cong 1/(g_m R_2 C_f R_1)$; $\omega_{p2}' \cong (g_m C_f)/(C_1 C_2 + C_f(C_1 + C_2))$

Pole Pair: $s^2 + (\omega_o/Q)s + \omega_o^2 = 0$; $Q \le 0.5 \Rightarrow$ real poles ; $Q > 1/\sqrt{2} \Rightarrow$ freq resp peaking

Power Amps: Class A: $\eta = (1/4)(\hat{V}_o/(IR_L))(\hat{V}_o/V_{CC})$ Class B: $\eta = (\pi/4)(\hat{V}_o/V_{CC})$; $P_{DN\_max} = V_{CC}^2/(\pi^2 R_L)$
Class AB: $i_n i_p = I_Q^2$

2-stage cmos opamp: $\omega_{p1} \approx (1/(R_1 G_{m2} R_2 C_c))$; $\omega_{p2} \approx (G_{m2}/C_2)$; $\omega_z \approx (1/(C_c((1/G_{m2}) - R)))$
$SR = I/C_c = \omega_t V_{ov1}$; will not SR limit if $\omega_t \hat{V}_o < SR$

MOS Transistor; CMOS basic parameters. Channel length = $0.18\mu m$

	V_t (V)	μC_{ox} $(\mu A/V^2)$	λ' $(\mu m/V)$	C_{ox} $(fF/\mu m^2)$	t_{ox} (nm)	L_{ov} (μm)	$\frac{C_{db0}}{W}$ $\left(\frac{fF}{\mu m}\right)$
NMOS	0.4	240	0.05	8.5	4	0.04	0.3
PMOS	-0.4	60	-0.05	8.5	4	0.04	0.3