University of Toronto

Term Test 2

Date - Nov 16, 2012 (3:15pm to 4:45pm)

Duration: 1.5 hrs

ECE331 — Analog Electronics
Lecturer - D. Johns

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

1. Equation sheet is on last page of test.
2. Unless otherwise stated, use transistor parameters on equation sheet.
3. Non-programmable calculator allowed; No other aids allowed
4. Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

Question	Mark
1	
2	
3	
4	
5	
6	
Total	

(max grade = 36)

Last Name: SOLUTIONS

First Name: ____________

Student #: ____________

[6] Question 1: Consider a first-order lowpass circuit with input $v_s(t)$, output $v_o(t)$, time-constant $\tau = 1\,\mu s$ and a dc gain of 2.

a) Give an expression for the transfer-function, $T(s)$ for this circuit.

$$T(s) = \frac{2}{1 + s/\omega_{3dB}} \qquad \omega_{3dB} = \frac{1}{\tau} = 1e6$$

$$= \frac{2}{1 + s/1e6}$$

$$\boxed{T(s) = \frac{2}{1 + s/(1e6)}}$$

b) What is the $f_{3\mathrm{dB}}$ frequency for $T(s)$.

$$f_{3dB} = \frac{\omega_{3dB}}{2\pi} = \frac{1}{2\pi\tau} = 159.2\ \mathrm{kHz}$$

$$\boxed{f_{3\mathrm{dB}} = 159.2\mathrm{kHz}}$$

c) Assuming $v_s(t)$ is a step input of 2V, in terms of τ, find the time, t_{set}, it takes for $v_o(t)$ to settle to within 0.1% of its final value.

$$y(t) = Y_\infty - (Y_\infty - Y_{0+})e^{-t/\tau}$$

$$\boxed{t_{\mathrm{set}} = 6.9\tau}$$

$Y_\infty = 2 \qquad Y_{0+} = 0$

$$y(t) = 2 - 2e^{-t/\tau} = 2(1 - e^{-t/\tau})$$

SETTLES TO WITHIN 0.1% WHEN

$$(1 - e^{-t/\tau}) = 0.999 \Rightarrow e^{-t/\tau} = 0.001$$

$$-t/\tau = -6.908$$

$$t = 6.908\,\tau$$

[6] Question 2: An amplifier has a voltage transfer-function of $T(s) = \dfrac{10^4 s}{(s+10)(s+10^3)}$.

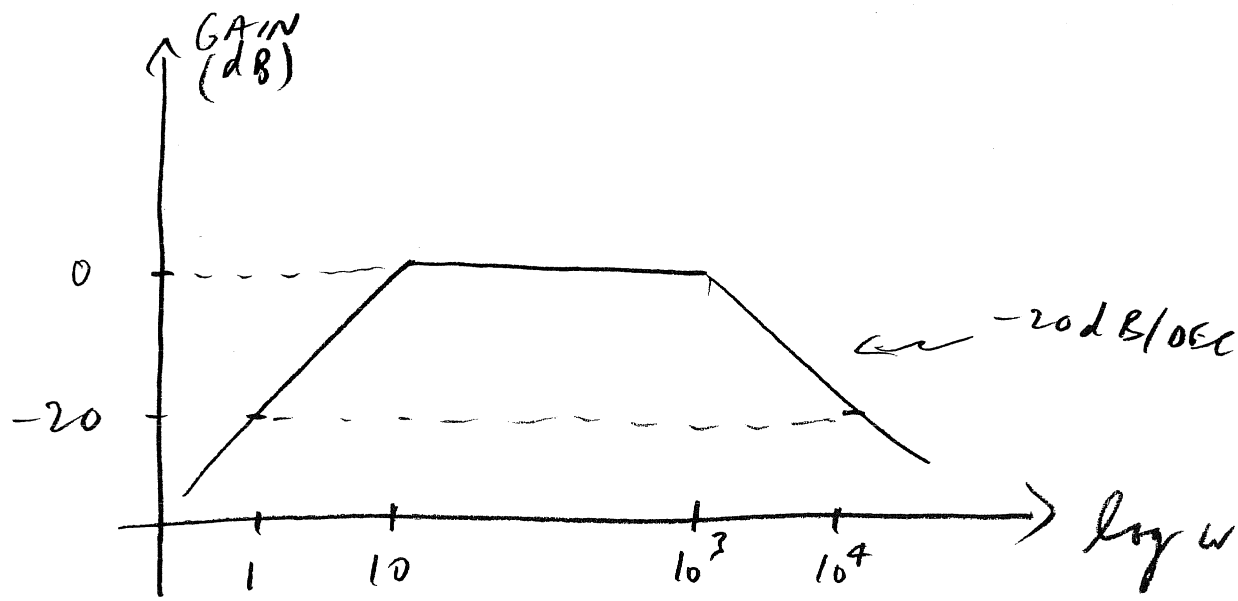
a) Convert $T(s)$ to a form suitable for a Bode plot (i.e. the denominator should have factors in the form $(1 + s/a)$).

$$T(s) = \frac{10^4 s}{(s+10)(s+10^3)} \times \frac{10^{-4}}{10^{-4}}$$

$$= \frac{s}{(1+s/10)(1+s/10^3)}$$

$$\boxed{T(s) = \frac{s}{(1+s/10)(1+s/10^3)}}$$

b) Draw a Bode plot for the magnitude response and use it to estimate the gain at frequency values of 1, 100, 10^4 rad/s.

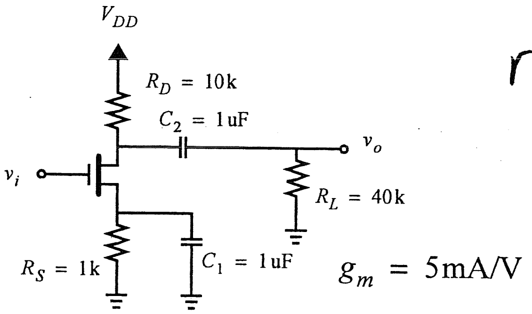


$|T(j1)| \simeq -20\ dB = 0.1$

$|T(j100)| \simeq 0\ dB = 1$

$|T(j10^4)| \simeq -20\ dB = 0.1$

[6] Question 3: Consider the amplifier circuit shown below.



$g_m = 5mA/V$

$r_o \to \infty$

$r_s = \frac{1}{g_m} = 200\,\Omega$

a) Find the low-freq small-signal gain and the high-freq small-signal gain.

LOW FREQ

$$\frac{v_o}{v_i} = -\frac{R_D \| R_L}{r_s + R_S} = \frac{-8}{1.2} = -6.67$$

$\left.\frac{v_o}{v_i}\right\|_{s=0}$	$= -6.67\ V/V$
$\left.\frac{v_o}{v_i}\right\|_{s\to\infty}$	$= -40\ V/V$

HIGH FREQ

$$\frac{v_o}{v_i} = -\frac{R_D \| R_L}{r_s} = \frac{-8}{0.2} = -40$$

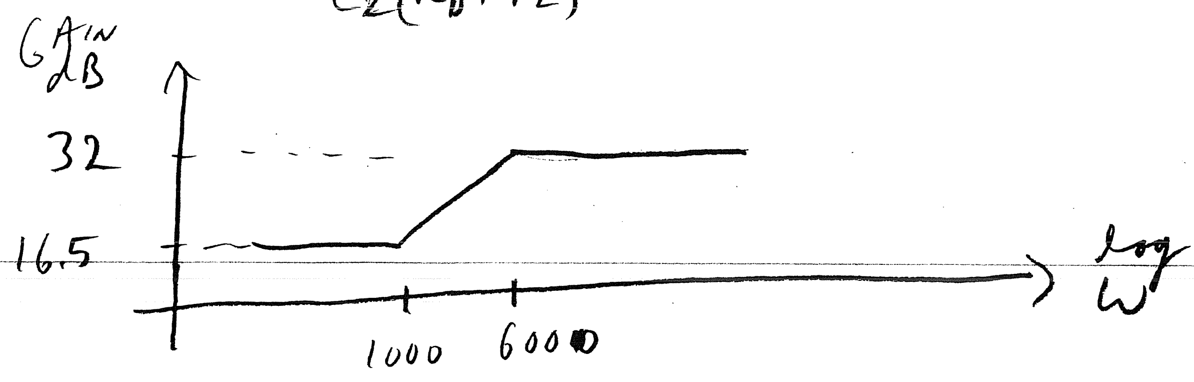
b) Given that the zero for this amplifier occurs at $\omega_z = 1/(R_S C_1) = 1k\text{ rad/s}$, sketch the bode plot for this amplifier using the dominant pole estimate for ω_L.

$$\omega_{L1} = \frac{1}{C_1 (R_1 \| r_s)} = 6000\text{ RAD/S}$$

$$\omega_{L2} = \frac{1}{C_2 (R_D + R_L)} = 20\text{ RAD/S}$$

$-6.67 \Rightarrow 16.5\,dB$

$-40 \Rightarrow 32\,dB$



[6] Question 4: Using the transistor parameters on the equation sheet, consider a transistor of size $W = 2\ \text{um}$, $L = 0.18\ \text{um}$.

a) Assuming the transistor is biased in the active region and assuming $V_{db} = 0$, find the values of C_{gs}, C_{gd} and C_{db} for the transistor (all in units of fF).

$$C_{gs} = \left(\tfrac{2}{3}\right) W L C_{ox} + W L_{ov} C_{ox}$$
$$= 2.72\ \text{fF}$$

$$C_{gd} = W L_{ov} C_{ox} = 0.68\ \text{fF}$$

$$C_{dB} = \frac{C_{dB0}}{W} \times W = 0.3 \times 2 = 0.6\ \text{fF}$$

LETTING $V_{dB} = 0 \Rightarrow C_{dB} = C_{dB0}$

$C_{gs} = 2.72\text{fF}$
$C_{gd} = 0.68\ \text{fF}$
$C_{db} = 0.6\ \text{fF}$

b) If $V_{ov} = 0.2\ \text{V}$, find the unity-gain freq of the transistor (do not ignore C_{gd}).

$$g_m = \mu_N C_{ox} \left(\frac{W}{L}\right) V_{ov} = (240e-6)\left(\frac{2}{0.18}\right)(0.2)$$
$$= 5.33e-4$$

$$f_t = \frac{g_m}{2\pi (C_{gs} + C_{gd})} = \frac{5.33e-4}{2\pi (2.72 + 0.68)(1e-15)}$$
$$= 24.97\ \text{GHz}$$

$\boxed{f_t = 25\ \text{GHz}}$

c) If the transistor width is doubled and the current through the transistor is also doubled, what is the new unity-gain frequency?

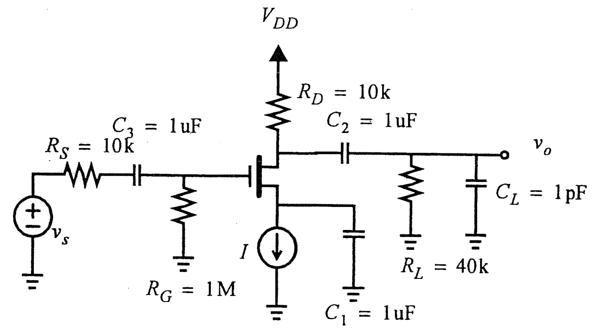
$$g_{m1} = \sqrt{2 \mu_N C_{ox} \left(\frac{W_1}{L}\right) I_{D1}}$$

IF $I_{D2} = 2 I_D$
$W_2 = 2W_1$

$$g_{m2} = 2 g_{m1}$$
$$\& \ (C_{gs2} + C_{gd2}) = 2(C_{gs1} + C_{gd1})$$
$$\Rightarrow f_{t2} = f_{t1}$$

NO CHANGE IN f_t

[6] Question 5: Consider the circuit shown below.



$C_{gs} = 1pF$
$C_{gd} = 0.2pF$
$C_{db} = 0.2pF$
$g_m = 5mA/V \Rightarrow r_s = \frac{1}{g_m} = 200$

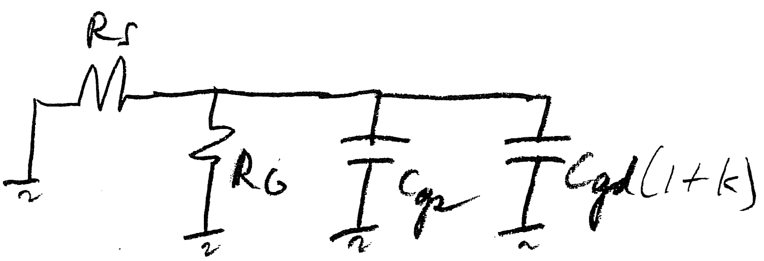
Estimate the 2 high frequency poles for this circuit (in Hz), f_{p1} and f_{p2}.

$$\frac{V_o}{V_g} = -\frac{R_D // R_L}{r_s} = -40\,V/V \equiv kk$$

$k = 40$

$\boxed{f_{p1} = 1.73\,MHz}$

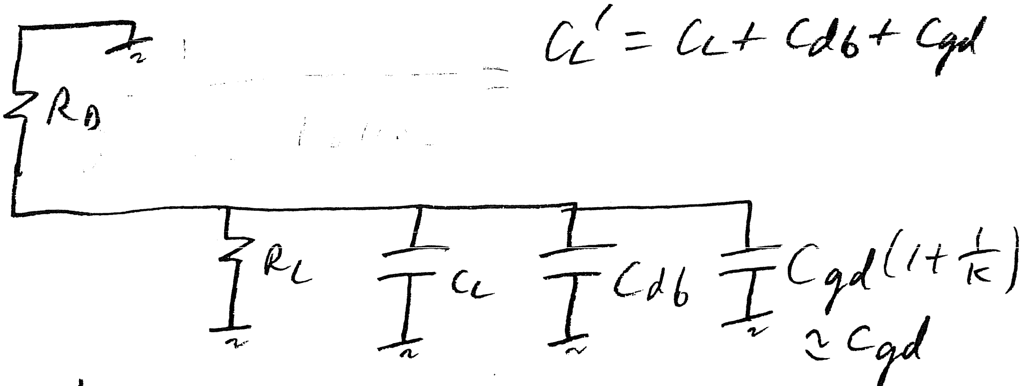
$\boxed{f_{p2} = 14.2\,MHz}$



$$f_{p1} = \frac{1}{2\pi\,(C_{gs} + C_{gd}(1+k))(R_G // R_S)}$$

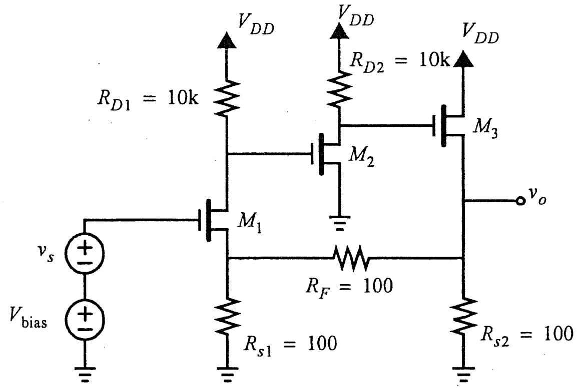
$\cong 1.73\,MHz$

$C_L' = C_L + C_{db} + C_{gd}$



$$f_{p2} = \frac{1}{2\pi\, C_L'\,(R_D // R_L)} = 14.2\,MHz$$

[6] Question 6: Consider the feedback circuit below. Find the loop gain, L.

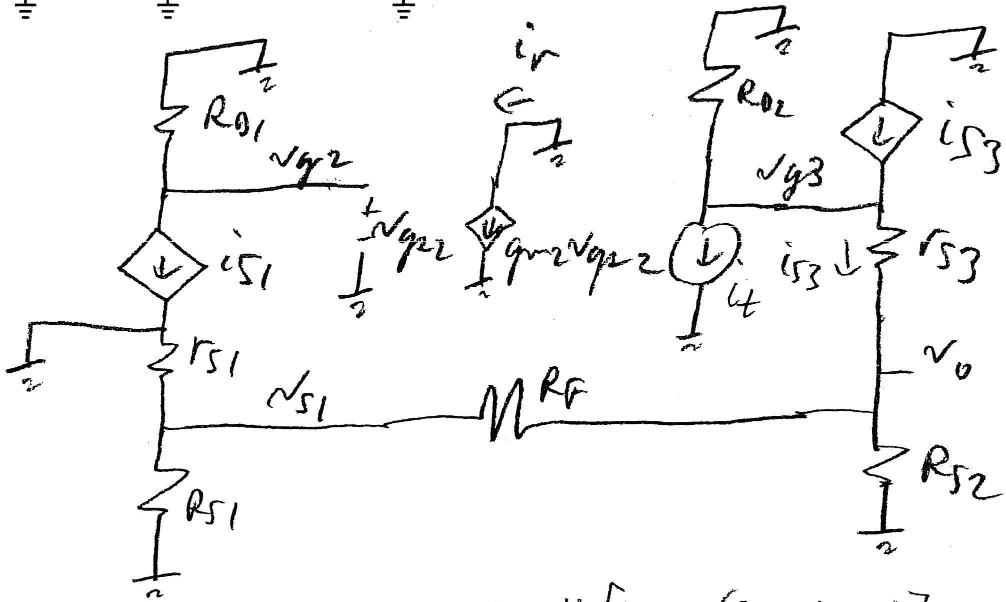


$g_{m1} = g_{m2} = g_{m3} = 5\text{mA/V}$

ignore r_0

$r_{s1} = r_{s2} = r_{s3} = 200$

$L = \boxed{238 \text{ V/V}}$



$$v_{g3} = -R_{D2}\, i_t = -10k\, i_t$$

$$v_o = \frac{R_{s2} \,||\, [R_F + (R_{s1} \,||\, r_{s1})]}{R_{s2} \,||\, [R_F + (R_{s1} \,||\, r_{s1})] + r_{s3}}\, v_{g3} = \frac{62.5}{62.5 + 200}\, v_{g3} = 0.238\, v_{g3}$$

$$v_{g2} = \frac{R_{D1}}{r_{s1}}\, v_{s1} = 50\, v_{s1} \qquad i_r = g_{m2} v_{g2} = 5e\text{-}3\, v_{g2}$$

$$v_{s1} = \frac{r_{s1} \,||\, R_{s1}}{(r_{s1} \,||\, R_{s1}) + R_F}\, v_o = 0.4\, v_o$$

$$L \equiv -\frac{i_r}{i_t} = (10k)(0.238)(0.4)(50)(5e\text{-}3) = \underline{238 \text{ V/V}}$$

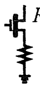
ECE331 Analog Electronics Equation Sheet

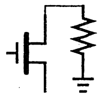
Constants: $k = 1.38 \times 10^{-23}\ \mathrm{JK}^{-1}$; $q = 1.602 \times 10^{-19}\ \mathrm{C}$; $V_T = kT/q \approx 26\mathrm{mV}$ at 300 °K ;
$\varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{F/m}$; $k_{\mathrm{ox}} = 3.9$; $C_{\mathrm{ox}} = (k_{\mathrm{ox}}\varepsilon_0)/t_{\mathrm{ox}}$

NMOS: $k_n = \mu_n C_{\mathrm{ox}}(W/L)$; $V_{tn} > 0$; $v_{\mathrm{DS}} \ge 0$; $v_{ov} = v_{GS} - V_{tn}$
(triode) $v_{\mathrm{DS}} \le v_{ov}$ (or $v_D < v_G - V_{tn}$) ; $i_D = k_n((v_{ov})v_{\mathrm{DS}} - (v_{\mathrm{DS}}^2/2))$
(active) $v_{\mathrm{DS}} \ge v_{ov}$; $i_D = 0.5k_n v_{ov}^{\ 2}(1 + \lambda v_{DS})$; $g_m = k_n V_{ov} = 2I_D/V_{ov} = \sqrt{2k_n I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda'|I_D)$

PMOS: $k_p = \mu_p C_{\mathrm{ox}}(W/L)$; $V_{tp} < 0$; $v_{SD} \ge 0$; $v_{ov} = v_{SG} - |V_{tp}|$
(triode) $v_{SD} \le v_{ov}$ (or $(v_D > v_G + |V_{tp}|)$) ; $i_D = k_p((v_{ov})v_{SD} - (v_{SD}^2/2))$
(active) $v_{\mathrm{DS}} \ge v_{ov}$; $i_D = 0.5k_p v_{ov}^{\ 2}(1 + |\lambda| v_{SD})$; $g_m = k_p V_{ov} = 2I_D/V_{ov} = \sqrt{2k_p I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda'|I_D)$

BJT: (active) $i_C = I_S e^{(v_{BE}/V_T)}(1 + (v_{CE}/V_A))$; $g_m = \alpha/r_e = I_C/V_T$; $r_\pi = \beta/g_m$; $r_o = |V_A|/I_C$
$i_C = \beta i_B$; $i_E = (\beta + 1)i_B$; $\alpha = \beta/(\beta + 1)$; $i_C = \alpha i_E$; $R_b = (\beta + 1)(r_e + R_E)$; $R_e = (R_B + r_\pi)/(\beta + 1)$

Cascode:  $R_{\mathrm{out}} = R_S + r_o + g_m r_o R_S \approx (1 + g_m R_S) r_o$
R_S

 R_D
$R_{in} = (R_D + r_o)/(1 + g_m r_o) \approx R_D/(g_m r_o) + 1/g_m$

Diff Pair: $A_d = g_m R_D$; $A_{CM} = -(R_D/(2R_{SS}))((\Delta R_D)/R_D)$; $A_{CM} = -(R_D/(2R_{SS}))((\Delta g_m)/g_m)$
$V_{os} = \Delta V_t$; $V_{os} = (V_{ov}/2)((\Delta R_D)/R_D)$; $V_{os} = (V_{ov}/2)((\Delta(W/L))/(W/L))$

1st order: step response $y(t) = Y_\infty - (Y_\infty - Y_{0+})e^{-t/\tau}$ unity gain freq for $T(s) = \dfrac{A_M}{1 + s/\omega_{3\mathrm{dB}}}$ $f_t \approx |A_M|\omega_{3\mathrm{dB}}$ when $A_M \gg 1$

Freq: for real axis poles/zeros $T(s) = k_{dc}\dfrac{(1 + s/z_1)(1 + s/z_2)\ldots(1 + s/z_m)}{(1 + s/\omega_1)(1 + s/\omega_2)\ldots(1 + s/\omega_n)}$
OTC estimate $f_H = 1/(2\pi\sum\tau_i)$; dominant pole estimate $f_H = 1/(2\pi\tau_{\max})$

Miller: $Z_1 = Z/(1 - K)$; $Z_2 = Z/(1 - 1/K)$

Mos caps: $C_{gs} = (2/3)WLC_{ox} + WL_{ov}C_{ox}$; $C_{gd} = WL_{ov}C_{ox}$; $C_{db} = C_{db0}/(\sqrt{1 + V_{db}/V_0})$
$f_t = g_m/(2\pi(C_{gs} + C_{gd}))$ assuming $C_{gd} \ll C_{gs}$ $f_t = (3\mu V_{ov})/(4\pi L^2)$

Feedback: $A_f = A/(1 + A\beta)$; $x_i = (1/(1 + A\beta))x_s$; $dA_f/A_f = (1/(1 + A\beta))dA/A$; $\omega_{Hf} = \omega_H(1 + A\beta)$; $\omega_{Lf} = \omega_L/(1 + A\beta)$
Loop Gain $L \equiv -s_r/s_t$

MOS Transistor; CMOS basic parameters. Channel length = $0.18\mu m$

	V_t (V)	μC_{ox} $(\mu A/V^2)$	λ' $(\mu\mathrm{m/V})$	C_{ox} $(\mathrm{fF}/\mu\mathrm{m}^2)$	t_{ox} (nm)	L_{ov} $(\mu\mathrm{m})$	$\frac{C_{db0}}{W}$ $\left(\frac{fF}{\mu\mathrm{m}}\right)$
NMOS	0.4	240	0.05	8.5	4	0.04	0.3
PMOS	-0.4	60	-0.05	8.5	4	0.02	0.3