

University of Toronto

Term Test 1

Date - Oct 21, 2013 (11:10pm to 12:00pm)

Duration: 50 min

ECE331 — Analog Electronics

Lecturer - D. Johns

ANSWER QUESTIONS ON THESE SHEETS USING BACKS IF NECESSARY

1. Equation sheet is on last page of test.
 2. Unless otherwise stated, use transistor parameters on equation sheet.
 3. Non-programmable calculator allowed; No other aids allowed
 4. Grading indicated by []. Attempt all questions since a blank answer will certainly get 0.
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Question	Mark
1	
2	
3	
4	
Total	

Last Name: SOLUTIONS

First Name: _____

Student #: _____

(max grade = 24)

[6] **Question 1:** Assume an nmos transistor (based on the 0.18 μ m parameters given on the equation sheet) with $W = 4\mu\text{m}$, $L = 0.5\mu\text{m}$ is biased with $V_{ov} = 0.4\text{V}$ and $V_{DS} = 1\text{V}$.

a) Find the r_o for this device.

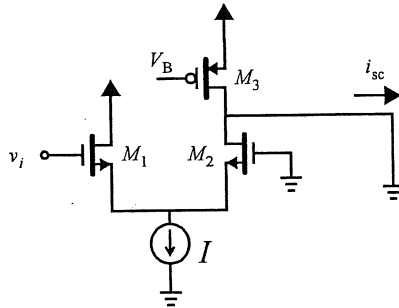
$$I_D = \frac{\mu_n C_{ox}}{2} \left(\frac{W}{L}\right) (V_{ov})^2 = \left(\frac{240 \text{e-6}}{2}\right) \left(\frac{4}{0.5}\right) (0.4)^2 = 154 \mu\text{A}$$

$$\lambda = \frac{\lambda'}{L} = \frac{0.05}{0.5} = 0.1 \text{ V}^{-1} \quad r_o = \frac{1}{\lambda I_D} = \underline{\underline{65.1 \text{ k}}}$$

b) If V_{DS} is increased by 0.5V, what is the corresponding change in I_D ?

$$\Delta I_D = \frac{\Delta V}{r_o} = \frac{0.5}{65.1 \text{ k}} = \underline{\underline{7.68 \mu\text{A}}}$$

[6] Question 3: Consider the circuit below.



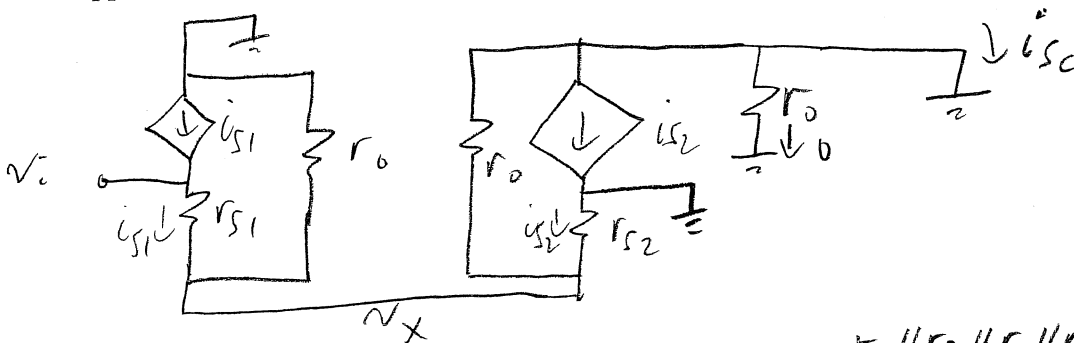
$g_{m1} = 1 \text{ mA/V}$
 $g_{m2} = 0.5 \text{ mA/V}$
 $g_{m3} = 1 \text{ mA/V}$

a) Find the small-signal short-circuit current gain, i_{sc}/v_i assuming all $r_o \rightarrow \infty$.

$r_{s1} = 1 \text{ k}$ $r_{s2} = 2 \text{ k}$ $i_{sc} = \frac{v_i}{r_{s1} + r_{s2}} = \frac{v_i}{3 \text{ k}}$

$\frac{i_{sc}}{v_i} = \frac{1}{3 \text{ k}} = \underline{\underline{0.333 \text{ mA/V}}}$

b) Find the small-signal gain, i_{sc}/v_i assuming $r_o = 20 \text{ k}$ for all transistors and making no approximations. Compare this result with that found in part a) in terms of percentage error.



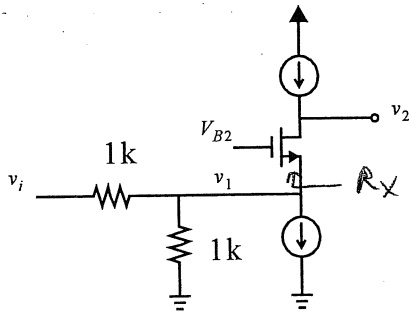
$$\frac{v_x - v_i}{r_{s1}} + \frac{v_x}{r_o} + \frac{v_x}{r_o} + \frac{v_x}{r_{s2}} = 0 \Rightarrow v_x = \frac{r_{s1} \parallel r_{s2} \parallel r_o \parallel r_o}{r_{s1}} v_i = 0.625 v_i$$

$$i_{sc} = \frac{v_x}{r_o} + \frac{v_x}{r_{s2}} = 0.625 v_i \left(\frac{1}{r_o} + \frac{1}{r_{s2}} \right) = \underline{\underline{0.3438 \text{ mA/V}}}$$

ERROR $\Rightarrow \frac{0.333 - 0.3438}{0.3438} \times 100 = -3^{rd}$ ERROR

$g_{m1} r_{o1} = 20$ $g_{m2} r_{o2} = 10$ (IF $g_{m1} r_{o1} = g_{m2} r_{o2}$ THEN ERROR = 0%) ASIDE

[6] **Question 4:** Consider the circuit shown below. All current sources and the transistors have the same output impedance of $r_o = 40\text{k}$. Also the transistors has $g_m = 1\text{mA/V}$. Estimate the small-signal gains, v_1/v_i , and v_2/v_1 . Make the assumption that $g_m r_o \gg 1$.



$v_1/v_i = 0.4 \text{ V/V}$
$v_2/v_1 = 20 \text{ V/V}$

$$R_x \approx \frac{1}{g_m} + \frac{r_o}{g_m r_o} = \frac{2}{g_m} = 2\text{ k}\Omega$$

$$\begin{aligned} \frac{v_1}{v_i} &\approx \frac{(R_x \parallel r_o \parallel 1\text{k})}{(R_x \parallel r_o \parallel 1\text{k}) + 1\text{k}} \approx \frac{(R_x \parallel 1\text{k})}{(R_x \parallel 1\text{k}) + 1\text{k}} \\ &= \frac{0.667}{0.667 + 1} = \underline{\underline{0.4 \text{ V/V}}} \end{aligned}$$

$$\frac{v_2}{v_1} \approx g_m (r_o \parallel r_o) = \underline{\underline{20 \text{ V/V}}}$$

Analog Electronics

Equation Sheet

Constants: $k = 1.38 \times 10^{-23} \text{ JK}^{-1}$; $q = 1.602 \times 10^{-19} \text{ C}$; $V_T = kT/q \approx 26 \text{ mV}$ at $300 \text{ }^\circ\text{K}$;

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}; k_{ox} = 3.9; C_{ox} = (k_{ox}\epsilon_0)/t_{ox}$$

NMOS: $k_n = \mu_n C_{ox}(W/L)$; $V_{in} > 0$; $v_{DS} \geq 0$; $v_{ov} = v_{GS} - V_{in}$

$$\text{(triode)} v_{DS} \leq v_{ov} \text{ (or } v_D < v_G - V_{in}); i_D = k_n((v_{ov})v_{DS} - (v_{DS}^2/2))$$

$$\text{(active)} v_{DS} \geq v_{ov}; i_D = 0.5k_n v_{ov}^2(1 + \lambda v_{DS}); g_m = k_n V_{ov} = 2I_D/V_{ov} = \sqrt{2k_n I_D}; r_s = 1/g_m; r_o = L/(\lambda' I_D)$$

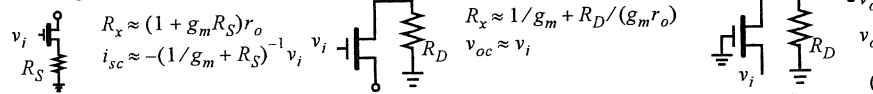
PMOS: $k_p = \mu_p C_{ox}(W/L)$; $V_{ip} < 0$; $v_{SD} \geq 0$; $v_{ov} = v_{SG} - |V_{ip}|$

$$\text{(triode)} v_{SD} \leq v_{ov} \text{ (or } v_D > v_G + |V_{ip}|); i_D = k_p((v_{ov})v_{SD} - (v_{SD}^2/2))$$

$$\text{(active)} v_{DS} \geq v_{ov}; i_D = 0.5k_p v_{ov}^2(1 + |\lambda|v_{SD}); g_m = k_p V_{ov} = 2I_D/V_{ov} = \sqrt{2k_p I_D}; r_s = 1/g_m; r_o = L/(\lambda' I_D)$$

BJT: (active) $i_C = I_S e^{(v_{BE}/V_T)}(1 + (v_{CE}/V_A))$; $g_m = \alpha/r_e = I_C/V_T$; $r_e = V_T/I_E$; $r_\pi = \beta/g_m$; $r_o = |V_A|/I_C$

$$i_C = \beta i_B; i_E = (\beta + 1)i_B; \alpha = \beta/(\beta + 1); i_C = \alpha i_E; R_b = (\beta + 1)(r_e + R_E); R_e = (R_B + r_\pi)/(\beta + 1)$$

Cascode:  $R_x \approx (1 + g_m R_S)r_o$; $i_{sc} \approx -(1/g_m + R_S)^{-1} v_i$; $R_x \approx 1/g_m + R_D/(g_m r_o)$; $v_{oc} \approx v_i$; $v_o/v_i \approx g_m(r_o \parallel R_D)$ (Approx due to $g_m r_o \gg 1$)

Diff Pair: $A_d = g_m R_D$; $A_{CM} = -(R_D/(2R_{SS}))((\Delta R_D)/R_D)$; $A_{CM} = -(R_D/(2R_{SS}))((\Delta g_m)/g_m)$

$$V_{os} = \Delta V_i; V_{os} = (V_{ov}/2)((\Delta R_D)/R_D); V_{os} = (V_{ov}/2)((\Delta(W/L))/(W/L))$$

1st order: step response $y(t) = Y_\infty - (Y_\infty - Y_{0+})e^{-t/\tau}$ unity gain freq for $T(s) = \frac{A_M}{1 + s/\omega_{3dB}}$ $f_i \approx |A_M|\omega_{3dB}$ when $A_M \gg 1$

Freq: for real axis poles/zeros $T(s) = k_{dc} \frac{(1 + s/z_1)(1 + s/z_2) \dots (1 + s/z_m)}{(1 + s/\omega_1)(1 + s/\omega_2) \dots (1 + s/\omega_n)}$

$$\text{OTC estimate } f_H = 1/(2\pi \sum \tau_i); \text{ dominant pole estimate } f_H = 1/(2\pi \tau_{max})$$

Miller: $Z_1 = Z/(1 - K)$; $Z_2 = Z/(1 - 1/K)$

Mos caps: $C_{gs} = (2/3)WLC_{ox} + WL_{ov}C_{ox}$; $C_{gd} = WL_{ov}C_{ox}$; $C_{db} = C_{db0}/(\sqrt{1 + V_{db}/V_0})$

$$f_i = g_m/(2\pi(C_{gs} + C_{gd})) \text{ assuming } C_{gd} \ll C_{gs} f_i = (3\mu V_{ov})/(4\pi L^2)$$

Feedback: $A_f = A/(1 + A\beta)$; $x_i = (1/(1 + A\beta))x_s$; $dA_f/A_f = (1/(1 + A\beta))dA/A$; $\omega_{Hf} = \omega_H(1 + A\beta)$; $\omega_{Lf} = \omega_L/(1 + A\beta)$

$$\text{Loop Gain } L \equiv -s_f/s_i; A_f = A_\infty(L/(1 + L)) + d/(1 + L); Z_{port} = Z_{p0}((1 + L_s)/(1 + L_o))$$

$$\text{PM} = \angle L(j\omega_1) + 180; \text{GM} = -|L(j\omega_{180})|_{dB}$$

$$\text{Pole Splitting } \omega_{p1}' \equiv 1/(g_m R_2 C_f R_1); \omega_{p2}' \equiv (g_m C_f)/(C_1 C_2 + C_f(C_1 + C_2))$$

Pole Pair: $s^2 + (\omega_o/Q)s + \omega_o^2 = 0$; $Q \leq 0.5 \Rightarrow$ real poles; $Q > 1/\sqrt{2} \Rightarrow$ freq resp peaking

Power Amps: Class A: $\eta = (1/4)(\hat{V}_o/(I_{RL}))(\hat{V}_o/V_{CC})$ Class B: $\eta = (\pi/4)(\hat{V}_o/V_{CC})$; $P_{DN_max} = V_{CC}^2/(\pi^2 R_L)$

$$\text{Class AB: } i_{n,i_p} = I_Q^2$$

2-stage cmos opamp: $\omega_{p1} \approx (1/(R_1 G_{m2} R_2 C_c))$; $\omega_{p2} \approx (G_{m2}/C_2)$; $\omega_z \approx (1/(C_c((1/G_{m2}) - R)))$

$$SR = I/C_c = \omega_i V_{ov1}; \text{ will not SR limit if } \omega_i \hat{V}_o < SR$$

MOS Transistor; CMOS basic parameters. Channel length = $0.18 \mu\text{m}$

	V_t (V)	μC_{ox} ($\mu\text{A}/\text{V}^2$)	λ' ($\mu\text{m}/\text{V}$)	C_{ox} ($\text{fF}/\mu\text{m}^2$)	t_{ox} (nm)
NMOS	0.4	240	0.05	8.5	4
PMOS	-0.4	60	-0.05	8.5	4