University of Toronto

Term Test 1

Date - Oct 12, 2012 (3:15pm to 4:45pm)

Duration: 1.5 hrs

ECE331 — Analog Electronics

Lecturer - D. Johns

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

1. Equation sheet is on last page of test.
2. Unless otherwise stated, use transistor parameters on equation sheet.
3. Non-programmable calculator allowed; No other aids allowed
4. Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

Question	Mark
1	
2	
3	
4	
5	
6	
Total	

(max grade = 36)

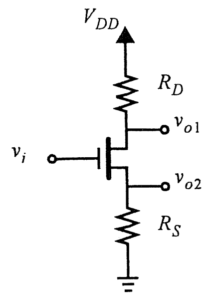
Last Name: SOLUTIONS

First Name: ____________________

Student #: ____________________

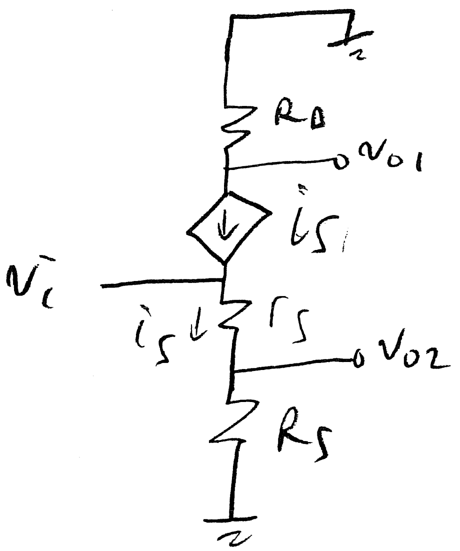
Last Name: ______________

[6] **Question 1:** For the nmos circuit below, draw the small-signal T-model and derive expressions for v_{o1}/v_i and v_{o2}/v_i in terms of R_D, R_S and $r_s = 1/g_m$. Assume $\lambda = 0$.



$$\frac{v_{o1}}{v_i} = -\frac{R_D}{r_s + R_S}$$

$$\frac{v_{o2}}{v_i} = \frac{R_S}{r_s + R_S}$$



$$i_s = \frac{v_i}{r_s + R_S}$$

$$v_{o1} = -i_s R_D$$

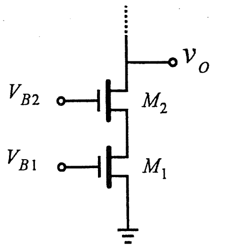
$$= -\frac{R_D}{r_s + R_S} v_i$$

$$\frac{v_{o1}}{v_i} = -\frac{R_D}{r_s + R_S}$$

$$v_{o2} = \frac{R_S}{r_s + R_S} v_i$$

$$\frac{v_{o2}}{v_i} = \frac{R_S}{r_s + R_S}$$

[6] Question 2: Consider the current mirrow below. If the current mirror is designed for a current of $20\mu A$, find the voltages V_{B1} and V_{B2} such that v_O can go as low as possible while the transistors remain in the active region. Find the minimum output voltage, v_{O_min}?



all $L = 0.5\mu m$
$W_1 = 2\mu m$
$W_2 = 1\mu m$
$I_D = 20\mu A$

V_{B1} =	0.604
V_{B2} =	1.293V
v_{O_min} =	0.493V

$$i_{D1} = \frac{\mu_n C_{ox}}{2}\left(\frac{W_1}{L}\right)(V_{OV1})^2$$

$$20\mu A = \left(\frac{240e-6}{2}\right)(4)V_{OV1}^2 \Rightarrow V_{OV1} = 0.204\ V$$

$$i_{D2} = i_{D1} \Rightarrow 20\mu A = \left(\frac{240e-6}{2}\right)(2)V_{OV2}^2 \Rightarrow V_{OV2} = 0.289\ V$$

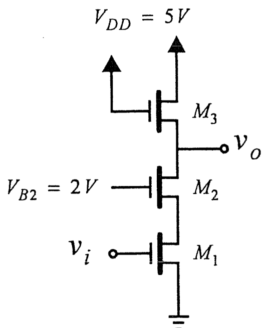
$$V_{B1} = V_{OV1} + V_{tn} = 0.204 + 0.4 = 0.604$$

$$\& \ \text{MIN}\ V_{D2} = V_{OV1} = 0.204\ V$$

$$V_{B2} = V_{OV2} + V_{tn} + V_{D2} = 0.893\ V$$

$$V_{O\_MIN} = V_{OV1} + V_{OV2} = 0.493\ V$$

[6] Question 3: Find the small-signal gain, v_o/v_i and output resistance R_o. (ignore r_o when finding i_{sc} but include r_o when finding R_o).



all $L = 0.5\mu m$
$W_1 = W_2 = 20\mu m$
$W_3 = 4\mu m$
v_i bias voltage is 1V

$\frac{v_o}{v_i} =$	$-2.16 V/V$
$R_o =$	750Ω

$V_{OV_1} = 1 - V_{tn} = 0.3V$

$i_{D1} = \left(\frac{\mu_N C_{OX}}{2}\right)\left(\frac{W_1}{L}\right)(V_{OV_1})^2 = 432\mu A$

$g_{m1} = \frac{2I_D}{V_{OV_1}} = 2.88 mA/V \Rightarrow r_{s1} = 347\Omega$

$g_{m2} = g_{m1}$ (SINCE $I_{D2} = I_{D1}$ & $W_2 = W_1$) & $g_{m3} = \sqrt{2\mu_n C_{ox}\left(\frac{W_3}{L}\right)I_D} = 1.29 mA$

$r_{s3} = 776\Omega$

$r_o = \frac{L}{\lambda' I_D} = \frac{0.5}{(0.05)(432\mu A)} = 23.1 k\Omega$

$i_{sc} = \frac{v_i}{r_{s1}} = \frac{v_i}{347}$

$R_o = r_{s3} \,||\, r_{o3} \,||\, [r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2}]$

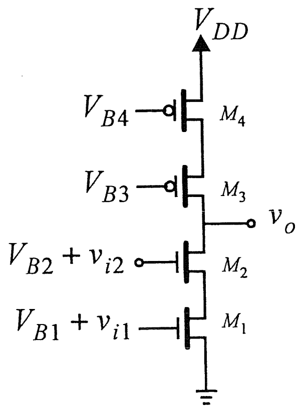
$= 776 \,||\, 23.1k \,||\, 1.58M = 750\Omega$

$v_o = -i_{sc} R_o = \frac{-R_o}{r_{s1}} v_i$

$\frac{v_o}{v_i} = -\frac{R_o}{r_{s1}} = \frac{-750\Omega}{347\Omega} = -2.16 V/V$

NOTE $\frac{v_o}{v_i} \approx \frac{-g_{m1}}{g_{m3}} = \sqrt{5}$

[6] Question 4: For the circuit below, all $V_{ov} = 200mV$ and all $I_D = 50\mu A$. The designer is concerned about noise on V_{B1} and V_{B2} bias lines. Find the gains v_o/v_{i1} and v_o/v_{i2}. (Include r_{o1} when finding i_{SC})



all $L = 0.5\mu m$

$\frac{v_o}{v_{i1}} =$	$-5100\ V/V$
$\frac{v_o}{v_{i2}} =$	$-50.5\ V/V$

$$g_{m1} = g_{m2} = g_{m3} = g_{m4} = \frac{2I_D}{V_{ov}} = 0.5mA/V$$

$$r_s = 2k\Omega = r_{s1} = r_{s2}$$

$$r_o = \frac{L}{\lambda' I_D} = 200k\Omega$$

$$R_o = R_{op} \| R_{on} \quad \& \quad R_{op} = R_{on} = r_o + r_o + g_m r_o r_o = 20.4M\Omega$$

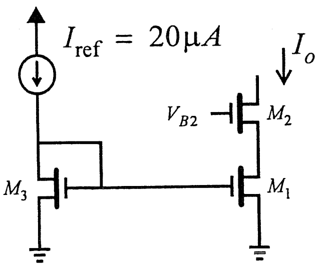
$$R_o = 10.2M\Omega$$

$$\frac{v_o}{v_{i1}} = -\frac{R_o}{r_{s1}} = \frac{-10.2M}{2k} = -5100\ V/V$$

$$i_{sc2} = \frac{v_{i2}}{r_{s2} + r_{o1}} \qquad v_o = -i_{sc2} R_o = -\frac{R_o}{r_{s2} + r_{o1}} v_{i2}$$

$$\frac{v_o}{v_{i2}} = -\frac{R_o}{r_{s2} + r_{o1}} = \frac{-10.2M}{202k} = -50.5\ V/V$$

[6] Question 5: Consider the current mirror shown below. Using small-signal analysis, estimate the change in I_o when there is a V_{tn} mismatch of 5mV between M_1 and M_3. Ignore r_o.



all $L = 0.5\mu m$

$W_1 = W_2 = W_3 = 2\mu m$

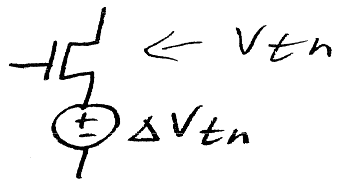
$\Delta I_o = 0.98\mu A$

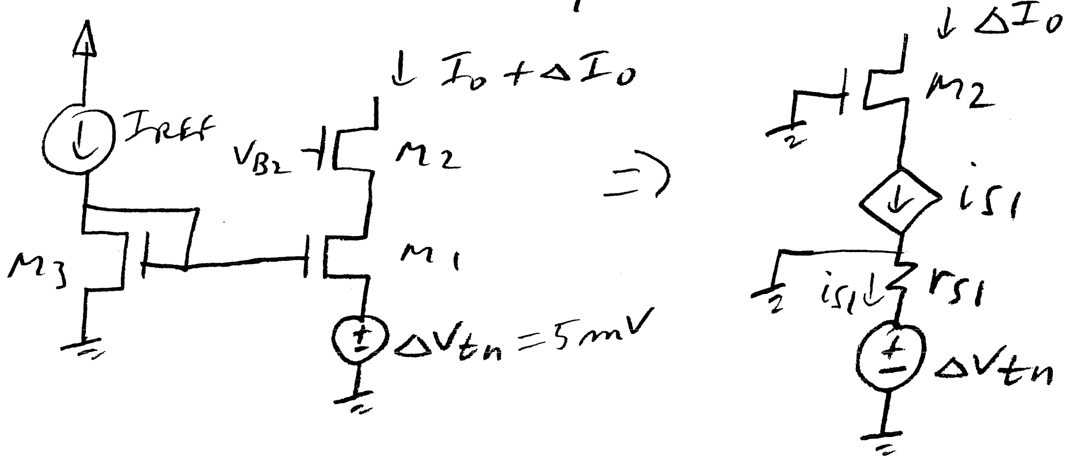
SINCE $W_1 = W_3 \Rightarrow I_O = 20\mu A$ IF $\Delta V_{tn} = 0$

$I_{D1} = \frac{\mu_N C_{ox}}{2}\left(\frac{W_1}{L}\right) V_{ov1}^2 \Rightarrow V_{ov1} = 0.204\,V$

$g_{m1} = \frac{2 I_D}{V_{ov1}} = 0.196\, mA/V = \frac{1}{r_{s1}}$

A TRANSISTOR WITH THRESHOLD $V_{tn} + \Delta V_{tn}$ EQUIVALENT TO



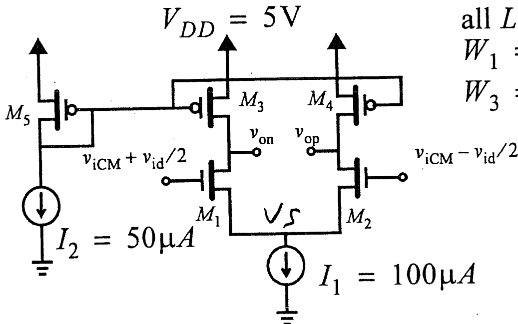


$\Delta I_o = i_{s1} = -\frac{\Delta V_{tn}}{r_{s1}} = -g_{m1}\Delta V_{tn} = -0.98\,\mu A$

SINCE V_{tn} MISMATCH <u>SIGN</u> UNKNOWN

ΔV_{tn} OF 5mV CAUSES $\Delta I_o = 0.98\,\mu A$

[6] Question 6: Consider the circuit below where $v_{iCM} = 2V$.



all $L = 0.5\mu m$
$W_1 = W_2 = 5\mu m$
$W_3 = W_4 = W_5 = 20\mu m$

$I_{D1} = 50\mu A$
$I_{D3} = 50\mu A$

a) What is the max and min for v_{op}?

$I_{D1} = \frac{\mu_N C_{ox}}{2}\left(\frac{W_1}{L}\right) V_{ov1}^2 \Rightarrow V_{ov1} = 0.204\ V$

$I_{D3} = \frac{\mu_P C_{ox}}{2}\left(\frac{W_3}{L}\right) V_{ov3}^2 \Rightarrow V_{ov3} = 0.204\ V$

$V_S = V_{icm} - V_{ov1} - V_{tn} = 1.4V$

$V_{op\_min} = V_S + V_{ov1} = 1.6\ V$

$V_{op\_max} = V_{DD} - V_{ov3} = 4.8V$

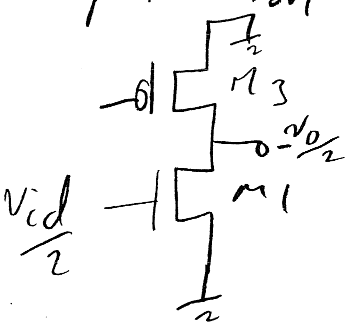
v_{op_min} =	$1.6\ V$
v_{op_max} =	$4.8\ V$

b) Find the gain, v_o/v_{id}.

$r_{o1} = \frac{L}{\lambda' I_D} = 200k\Omega = r_{o3}$

$g_{m1} = \frac{2I_D}{V_{ov1}} = 0.49\ mA/V \quad r_{s1} = 2.04\ k\Omega$

v_o/v_{id} = 49 V/V



HALF CIRCUIT

$-\frac{v_o}{2} = -g_{m1}(r_{o1} \| r_{o3})\frac{v_{id}}{2}$

$\frac{v_o}{v_{id}} = g_{m1}(r_{o1} \| r_{o2}) = 49\ V/V$

ECE331 Analog Electronics Equation Sheet

Constants: $k = 1.38 \times 10^{-23}\ \mathrm{JK}^{-1}$; $q = 1.602 \times 10^{-19}$ C ; $V_T = kT/q \approx 26\mathrm{mV}$ at 300 °K ;
$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m ; $k_{ox} = 3.9$; $C_{ox} = (k_{ox}\varepsilon_0)/t_{ox}$

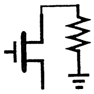
NMOS: $k_n = \mu_n C_{ox}(W/L)$; $V_{tn} > 0$; $v_{DS} \geq 0$; $v_{ov} = v_{GS} - V_{tn}$
(triode) $v_{DS} \leq v_{ov}$ (or $v_D < v_G - V_{tn}$) ; $i_D = k_n((v_{ov})v_{DS} - (v_{DS}^2/2))$
(active) $v_{DS} \geq v_{ov}$; $i_D = 0.5k_n v_{ov}^2(1 + \lambda v_{DS})$; $g_m = k_n V_{ov} = 2I_D/V_{ov} = \sqrt{2k_n I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda'|I_D)$

PMOS: $k_p = \mu_p C_{ox}(W/L)$; $V_{tp} < 0$; $v_{SD} \geq 0$; $v_{ov} = v_{SG} - |V_{tp}|$
(triode) $v_{SD} \leq v_{ov}$ (or $(v_D > v_G + |V_{tp}|)$) ; $i_D = k_p((v_{ov})v_{SD} - (v_{SD}^2/2))$
(active) $v_{DS} \geq v_{ov}$; $i_D = 0.5k_p v_{ov}^2(1 + |\lambda| v_{SD})$; $g_m = k_p V_{ov} = 2I_D/V_{ov} = \sqrt{2k_p I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda'|I_D)$

BJT: (active) $i_C = I_S e^{(v_{BE}/V_T)}(1 + (v_{CE}/V_A))$; $g_m = \alpha/r_e = I_C/V_T$; $r_\pi = \beta/g_m$; $r_o = |V_A|/I_C$
$i_C = \beta i_B$; $i_E = (\beta+1)i_B$; $\alpha = \beta/(\beta+1)$; $i_C = \alpha i_E$; $R_b = (\beta+1)(r_e + R_E)$; $R_e = (R_B + r_\pi)/(\beta+1)$

Cascode:

$R_{out} = R_S + r_o + g_m r_o R_S \approx (1 + g_m R_S) r_o$



$R_{in} = (R_D + r_o)/(1 + g_m r_o) \approx R_D/(g_m r_o) + 1/g_m$

Diff Pair: $A_d = g_m R_D$; $A_{CM} = -(R_D/(2R_{SS}))((\Delta R_D)/R_D)$; $A_{CM} = -(R_D/(2R_{SS}))((\Delta g_m)/g_m)$
$V_{os} = \Delta V_t$; $V_{os} = (V_{ov}/2)((\Delta R_D)/R_D)$; $V_{os} = (V_{ov}/2)((\Delta(W/L))/(W/L))$

<u>MOS Transistor;</u> CMOS basic parameters. Channel length = $0.18\mu m$

	V_t (V)	μC_{ox} $(\mu A/V^2)$	λ' (µm/V)	C_{ox} (fF/µm^2)	t_{ox} (nm)
NMOS	0.4	240	0.05	8.5	4
PMOS	-0.4	60	-0.05	8.5	4