University of Toronto

Final Exam

Date - Dec 16, 2013

Duration: 2.5 hrs

ECE331 — Electronic Circuits

Lecturer - D. Johns

ANSWER QUESTIONS ON THESE SHEETS **USING BACKS IF NECESSARY**

1. Equation sheet is on last page of test.
2. Unless otherwise stated, use transistor parameters on equation sheet and assume $g_m r_o \gg 1$.
3. Non-programmable calculator allowed; No other aids allowed
4. Grading indicated by [ ]. Attempt all questions since a blank answer will certainly get 0.

Last Name: SOLUTIONS

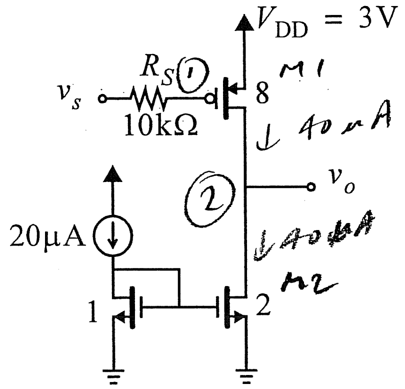
First Name: ____________

Student #: ____________

Question	Mark
1	
2	
3	
4	
5	
6	
Total	

(max grade = 36)

[6] **Question 1:** Consider the circuit below where all transistors are in the active region. The numbers beside the transistors indicate the transistor width (in μm).



All $L = 0.18\mu$m

$r_o = \frac{L}{(\lambda') I_D} = \frac{3.6}{I_D}$

a) Find the small-signal gain v_o/v_s

$r_{o1} = r_{o2} = 90k$

$\boxed{v_o/v_s = -20.8}$ V/V

$g_{m1} = \sqrt{2 \mu_p C_{ox} \left(\frac{W}{L}\right) I_D} = 462 \mu A/V$

$\frac{v_o}{v_s} = -g_{m1}(r_{o1} \| r_{o2}) = -(462e{-6})\left(\frac{90k}{2}\right) = -20.8$ V/V

$K = -20.8$

b) Estimate the 3db frequency cutoff, f_{3dB}. For C_{db} values assume $V_{db} = 0$.

NODE 1 $C_{gs} = \frac{2}{3} W L C_{ox} + W L_{ov} C_{ox} = 10.9$ fF

$\boxed{f_{3dB} = 393}$ MHz

$C_{gd1} = W L_{ov} C_{ox} = (8)(0.02)(8.5e{-15}) = 1.36$ fF

$\omega_{P1} = \frac{1}{R_S[C_{gs} + (1-K) C_{gd}]} = \frac{1}{(10k)(40.5 fF)} = 2.47e9$ RAD/S $= 393$ MHz

NODE 2 $C_d = C_{d1} + C_{d2} = (W_1 + W_2)\left(\frac{C_{db0}}{W}\right) = (10)(0.3 fF/\mu m)$
$= 3$ fF

$\omega_{P2} = \frac{1}{(r_{o1} \| r_{o2}) C_d} = \frac{1}{(45k) 3fF} = 7.4e9$ RAD/S $= 1.2$ GHz

[6] Question 2:

a) For a Mosfet transistor, show that $f_t \approx \frac{3\mu_n V_{ov}}{4\pi L^2}$ assuming $C_{gs} \gg C_{gd}$ and the overlap component of C_{gs} is negligibly small.

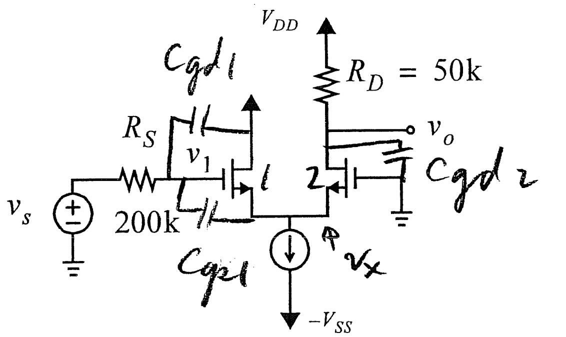
$$f_t = \frac{g_m}{2\pi(C_{gs}+C_{gd})} \approx \frac{g_m}{2\pi C_{gs}}$$

$$g_m = \mu_N C_{ox}\left(\frac{W}{L}\right)V_{ov}$$

$$C_{gs} = \left(\frac{2}{3}\right) W L C_{ox}$$

$$f_t = \frac{\mu_N \cancel{C_{ox}}\left(\frac{\cancel{W}}{L}\right)V_{ov}}{2\pi\left(\frac{2}{3}\right)\cancel{W} L \cancel{C_{ox}}} = \frac{3}{4}\,\frac{\mu_N V_{ov}}{\pi L^2}$$

b) Consider the circuit below, only consider the capacitors C_{gs} and C_{gd}.



for each transistor
$g_m = 1$ mA/V
$C_{gs} = C_{gd} = 1$ pF
$r_o \to \infty$

Find the pole due to the node v_1 and also find the pole due to the node v_o.

$$K \equiv \frac{v_x}{v_1} = \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m2}}+\frac{1}{g_{m1}}} = 0.5$$

$\omega_{p1} =$ 3.33e6	RAD/S
$\omega_{p2} =$ 20e6	RAD/S

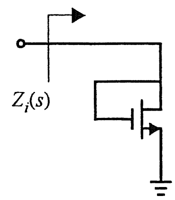
v_1 MILLER on $C_{gs1} \Rightarrow C_g' = C_{gs1}(1-k)$
$= 0.5\, C_{gs}$

$$W_{p1} = \frac{1}{R_S(C_g' + C_{gd})} = \frac{1}{(200k)(1.5pF)} = 3.33e6$$

V_o

$$W_{p2} = \frac{1}{R_D C_{gd2}} = \frac{1}{(50k)(1pF)} = 20e6$$

[6] Question 3: Consider the transistor below where it is biased such that $V_{ov} = 0.2\text{V}$. Including the effects of C_{gs} and C_{gd}, find the frequency, f_{45}, where the impedance has a phase angle of -45° (in Hz). Ignore r_o.



$V_{ov} = 0.2\text{V}$
$W = 3\text{um}$
$L = 0.3\text{um}$
$r_o \rightarrow \infty$

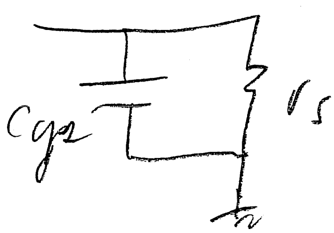
$\boxed{f_{45} = 12.5 \text{ GHz}}$

$$I_D = \frac{\mu_N C_{ox}}{2}\left(\frac{W}{L}\right)V_{OV}^2 = \left(\frac{240e\text{-}6}{2}\right)\left(\frac{3}{0.3}\right)(0.2)^2 = 48\,\mu A$$

$$g_m = \frac{2I_D}{V_{OV}} = 480\,\mu A/V \qquad r_s = \frac{1}{g_m} = 2.08\,k\Omega$$

$$C_{gs} = \left(\frac{2}{3}\right)W L C_{ox} + W L_{ov} C_{ox} = 6.12\,fF$$

C_{gd} UNIMPORTANT SINCE GATE DRAIN SHORTED



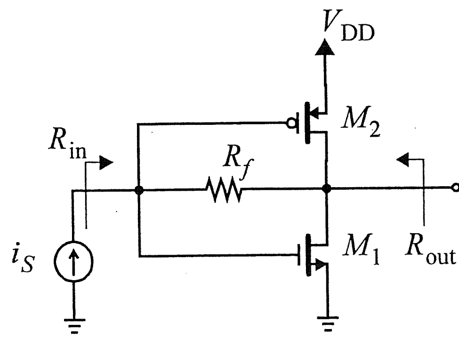
$$Z_{IN} = r_s \,//\, \frac{1}{sC_{gs}}$$

$$= \frac{r_s}{1 + sC_{gs}r_s}$$

$$\angle Z_{IN} = -45^{\circ} \quad \text{AT} \quad \omega_{45} = \frac{1}{C_{gs}r_s} = 7.86e10 \text{ RAD/S}$$

$$f_{45} = \frac{\omega_{45}}{2\pi} = 12.5 \text{ GHz}$$

[6] Question 4: Consider the circuit shown below.



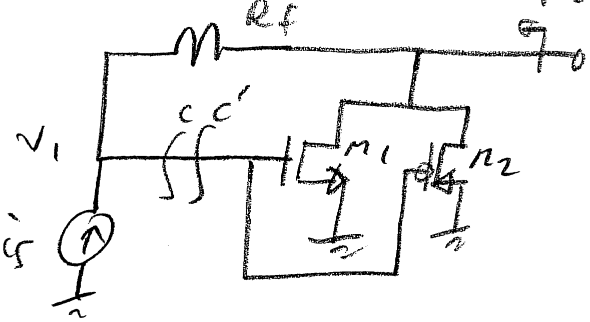
$g_{m1} = g_{m2} = 100\text{uA/V}$ $\quad A_{cl} \equiv v_O/i_S$
$r_{o1} = r_{o2} = 100\text{k}$
$R_f = 100\text{k}$

Hint: transconductance gm1 is parallel to gm2

a) Find L, A_∞ and d.

REDRAW SCHEMATIC (SMALL-SIGNAL)

$L = 10$ V/V
$A_\infty = -100k$ Ω
$d = 50k$ Ω



$L = (g_{m1} + g_{m2})(r_{o1} \| r_{o2})$
$= 10$ V/V

$A_\infty \Rightarrow v_1 \to 0 \Rightarrow v_o = -R_f i_s \Rightarrow A_\infty = -R_f = -100k$

$d = (r_{o1} \| r_{o2}) = 50k$

b) Find v_o/i_s, R_{in} and R_{out}.

v_o/i_s = -86.4k Ω
R_{in} = 13.6k Ω
R_{out} = 4.55k Ω

$$\frac{v_o}{i_s} = A_\infty\left(\frac{L}{1+L}\right) + d\left(\frac{1}{1+L}\right)$$

$$= -100k\left(\frac{10}{11}\right) + 50k\left(\frac{1}{11}\right)$$

$$= -86.4K$$

$$R_{IN} = \left[R_F + (r_{o1} \| r_{o2})\right] / (1+L_o)$$

$L_S = 0$

$L_o = L$

$$= 13.6k$$

$$R_{OUT} = \frac{(r_{o1} \| r_{o2})}{(1+L_o)}$$

$L_S = 0$

$L_o = L$

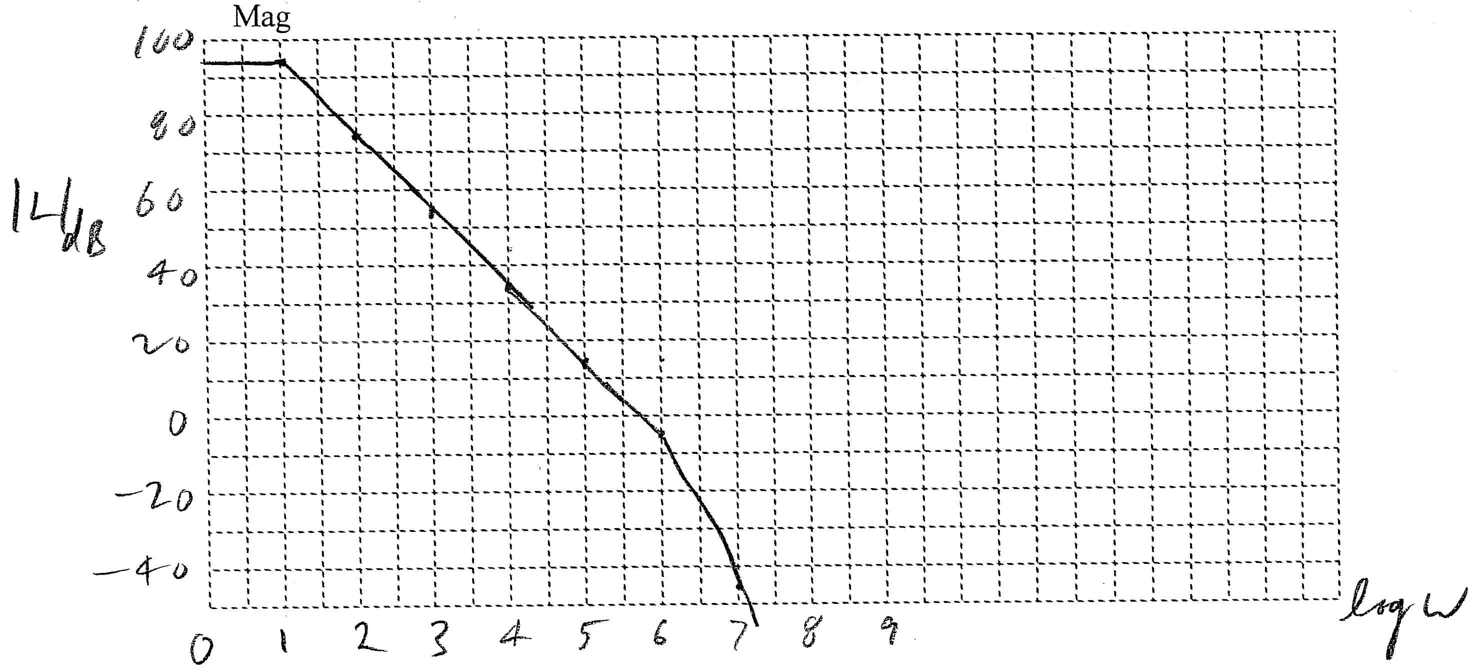
$$= 4.55K$$

[6] **Question 5:** Assume the loop gain for a feedback system is found to be the following.

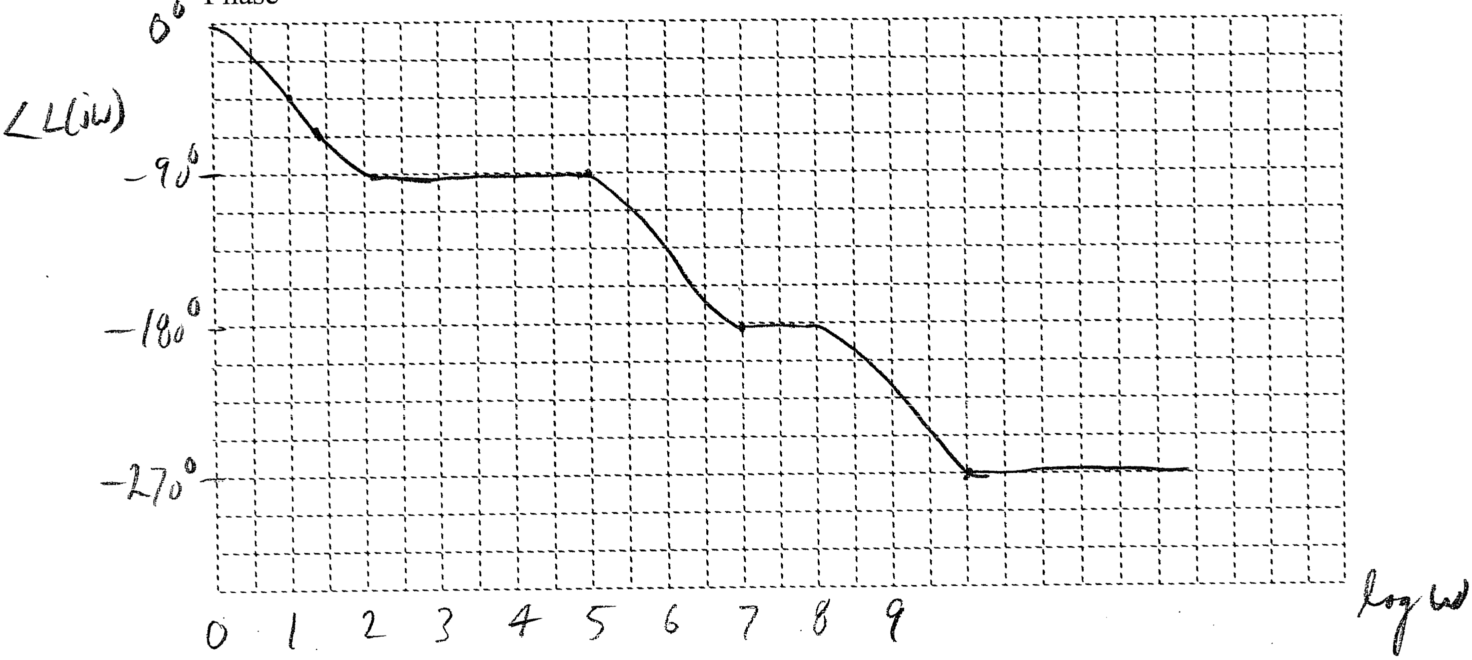
$$L(s) = \frac{5 \times 10^4}{(1 + s/\omega_{p1})(1 + s/\omega_{p2})(1 + s/\omega_{p3})} \quad \text{where } \omega_{p1} = 10^1,\ \omega_{p2} = 10^6,\ \omega_{p3} = 10^9.$$

a) Sketch the Bode plot for the above loop gain (both mag and phase)

Mag



Phase



b) Estimate the phase-margin (PM) for the above loop gain. Hint, the unity gain freq is much greater than ω_{p1} and much less than ω_{p3}.

$$L(s) = \frac{5\times10^4}{(1+\frac{s}{10})(1+\frac{s}{10^6})(1+\frac{s}{10^9})}$$

$$\boxed{PM = 66^\circ}$$

$|L(j\omega_t)|=1$ & $10 \ll \omega_t \ll 10^9$ GIVEN

$$|L(j\omega_t)| \approx \frac{5e4}{\left|\frac{\omega_t}{10}\right|\left|1+\frac{j\omega_t}{10^6}\right|} = 1 \Rightarrow \frac{(5e4)^2}{\left(\frac{\omega_t}{10}\right)^2\left(1+\left(\frac{\omega_t}{10^6}\right)^2\right)} = 1$$

$$\frac{10^2\,10^{12}(5e4)^2}{\omega_t^2(10^{12}+\omega_t^2)} = 1 \Rightarrow \omega_t^2 + 1e12\,\omega_t^2 - 2.5e23 = 0 \Rightarrow \omega_t^2 = 2.07e11$$

$$\omega_t = 4.6e5$$

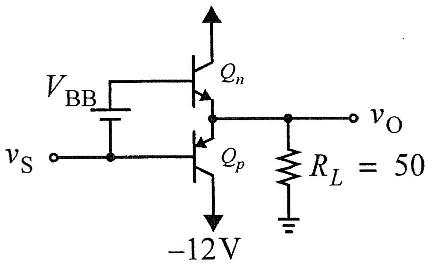
$$\angle L(j\omega_t) \approx -90^\circ - \tan^{-1}\left(\frac{4.6e6}{1e6}\right) = -90^\circ - 24.5^\circ = -114.5^\circ$$

$$PM = \angle L(j\omega_t) + 180^\circ = 66^\circ$$

c) Estimate the time-constant for the settling behaviour of this feedback system (assuming it can be modeled as a first-order system).

$$\tau \approx \frac{1}{\omega_t} = \frac{1}{2.88e5} = \underline{3.5\,\mu s}$$

[6] Question 6: Consider a class AB BJT output stage shown below. Assume transistors Q_n and Q_p are matched with $I_s = 10^{-13}$ A, β is large, and the output is sinusoidal with a maximum amplitude of 10V.



Assume $V_T = 25$mV

$$i_C = I_S e^{\frac{V_{BE}}{V_T}}$$

$$V_T = V_T \ln\left(\frac{i_C}{I_S}\right)$$

a) Find the value of V_{BB} such that the quiescent current is 5% of the maximum load current.

$$I_{LMAX} = \frac{10}{50} = 200\,mA \Rightarrow I_Q = 5\% \times 200\,mA$$

$$I_Q = 10\,mA$$

$\boxed{V_{BB} = 1.266\,V}$

$$\frac{V_{BB}}{2} = V_T \ln\left(\frac{I_Q}{I_S}\right) \Rightarrow V_{BB} = 2 \times 25\,mV \ln\left(\frac{10e-3}{1e-13}\right) = 1.266\,V$$

b) For an output voltage of -5V, estimate i_n, i_p, and v_S.

$$i_n i_p = I_Q^2 \qquad I_Q = 10\,mA$$

FOR $V_o = -5V$ $\qquad i_n$ SMALL

$$i_p \cong \frac{5}{50} = 100\,mA$$

$$i_n = \frac{I_Q^2}{i_p} = 1\,mA$$

$i_n =$	$1\,mA$
$i_p =$	$100\,mA$
$v_S =$	$-5.69\,V$

$$V_{EBP} = V_T \ln\left(\frac{i_P}{I_S}\right) = 25\,mV \ln\left(\frac{100e-3}{1e-13}\right) = 0.69V$$

$$V_S = V_o - V_{EBP} = -5 - 0.69V = -5.69V$$

Last Name: ____________

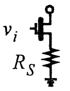
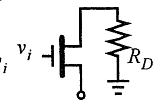
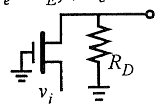
Analog Electronics **Equation Sheet**

Constants: $k = 1.38 \times 10^{-23}\ \mathrm{JK}^{-1}$; $q = 1.602 \times 10^{-19}$ C ; $V_T = kT/q \approx 26\mathrm{mV}$ at 300 °K ;
$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m ; $k_{ox} = 3.9$; $C_{ox} = (k_{ox}\varepsilon_0)/t_{ox}$

NMOS: $k_n = \mu_n C_{ox}(W/L)$; $V_{tn} > 0$; $v_{DS} \ge 0$; $v_{ov} = v_{GS} - V_{tn}$
(triode) $v_{DS} \le v_{ov}$ (or $v_D < v_G - V_{tn}$) ; $i_D = k_n((v_{ov})v_{DS} - (v_{DS}^2/2))$
(active) $v_{DS} \ge v_{ov}$; $i_D = 0.5k_n v_{ov}^2(1 + \lambda v_{DS})$; $g_m = k_n V_{ov} = 2I_D/V_{ov} = \sqrt{2k_n I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda'|I_D)$

PMOS: $k_p = \mu_p C_{ox}(W/L)$; $V_{tp} < 0$; $v_{SD} \ge 0$; $v_{ov} = v_{SG} - |V_{tp}|$
(triode) $v_{SD} \le v_{ov}$ (or $(v_D > v_G + |V_{tp}|)$) ; $i_D = k_p((v_{ov})v_{SD} - (v_{SD}^2/2))$
(active) $v_{DS} \ge v_{ov}$; $i_D = 0.5k_p v_{ov}^2(1 + |\lambda| v_{SD})$; $g_m = k_p V_{ov} = 2I_D/V_{ov} = \sqrt{2k_p I_D}$; $r_s = 1/g_m$; $r_o = L/(|\lambda'|I_D)$

BJT: (active) $i_C = I_S e^{(v_{BE}/V_T)}(1 + (v_{CE}/V_A))$; $g_m = \alpha/r_e = I_C/V_T$; $r_e = V_T/I_E$; $r_\pi = \beta/g_m$; $r_o = |V_A|/I_C$
$i_C = \beta i_B$; $i_E = (\beta+1)i_B$; $\alpha = \beta/(\beta+1)$; $i_C = \alpha i_E$; $R_b = (\beta+1)(r_e + R_E)$; $R_e = (R_B + r_\pi)/(\beta+1)$

Cascode:  $R_x \approx (1 + g_m R_S)r_o$; $i_{sc} \approx -(1/g_m + R_S)^{-1} v_i$
 $R_x \approx 1/g_m + R_D/(g_m r_o)$; $v_{oc} \approx v_i$
 $v_o/v_i \approx g_m(r_o \| R_D)$ (Approx due to $g_m r_o \gg 1$)

Diff Pair: $A_d = g_m R_D$; $A_{CM} = -(R_D/(2R_{SS}))((\Delta R_D)/R_D)$; $A_{CM} = -(R_D/(2R_{SS}))((\Delta g_m)/g_m)$
$V_{os} = \Delta V_t$; $V_{os} = (V_{ov}/2)((\Delta R_D)/R_D)$; $V_{os} = (V_{ov}/2)((\Delta(W/L))/(W/L))$

1st order: step response $y(t) = Y_\infty - (Y_\infty - Y_{0+})e^{-t/\tau}$ unity gain freq for $T(s) = \dfrac{A_M}{1 + s/\omega_{3\mathrm{dB}}}$ $f_t \approx |A_M|\omega_{3\mathrm{dB}}$ when $A_M \gg 1$

Freq: for real axis poles/zeros $T(s) = k_{dc}\dfrac{(1+s/z_1)(1+s/z_2)\ldots(1+s/z_m)}{(1+s/\omega_1)(1+s/\omega_2)\ldots(1+s/\omega_n)}$
OTC estimate $f_H = 1/(2\pi\sum\tau_i)$; dominant pole estimate $f_H = 1/(2\pi\tau_{\max})$

Miller: $Z_1 = Z/(1-K)$; $Z_2 = Z/(1-1/K)$

Mos caps: $C_{gs} = (2/3)WLC_{ox} + WL_{ov}C_{ox}$; $C_{gd} = WL_{ov}C_{ox}$; $C_{db} = C_{db0}/(\sqrt{1 + V_{db}/V_0})$
$f_t = g_m/(2\pi(C_{gs} + C_{gd}))$ assuming $C_{gd} \ll C_{gs}$ $f_t = (3\mu V_{ov})/(4\pi L^2)$

Feedback: $A_f = A/(1+A\beta)$; $x_i = (1/(1+A\beta))x_s$; $dA_f/A_f = (1/(1+A\beta))dA/A$; $\omega_{Hf} = \omega_H(1+A\beta)$; $\omega_{Lf} = \omega_L/(1+A\beta)$
Loop Gain $L \equiv -s_r/s_t$; $A_f = A_\infty(L/(1+L)) + d/(1+L)$; $Z_{\mathrm{port}} = Z_{p0}((1+L_S)/(1+L_O))$
PM $= \angle L(j\omega_1) + 180$; GM $= -|L(j\omega_{180})|_{\mathrm{dB}}$
Pole Splitting $\omega_{p1}' \cong 1/(g_m R_2 C_f R_1)$; $\omega_{p2}' \cong (g_m C_f)/(C_1 C_2 + C_f(C_1 + C_2))$

Pole Pair: $s^2 + (\omega_o/Q)s + \omega_o^2 = 0$; $Q \le 0.5 \Rightarrow$ real poles ; $Q > 1/\sqrt{2} \Rightarrow$ freq resp peaking

Power Amps: Class A: $\eta = (1/4)(\hat{V}_o/(IR_L))(\hat{V}_o/V_{CC})$ Class B: $\eta = (\pi/4)(\hat{V}_o/V_{CC})$; $P_{\mathrm{DN\_max}} = V_{CC}^2/(\pi^2 R_L)$
Class AB: $i_n i_p = I_Q^2$

2-stage cmos opamp: $\omega_{p1} \approx (1/(R_1 G_{m2} R_2 C_c))$; $\omega_{p2} \approx (G_{m2}/C_2)$; $\omega_z \approx (1/(C_c((1/G_{m2}) - R)))$
$SR = I/C_c = \omega_t V_{ov1}$; will not SR limit if $\omega_t \hat{V}_o < SR$

MOS Transistor; CMOS basic parameters. Channel length = $0.18\mu m$

	V_t (V)	μC_{ox} $(\mu A/V^2)$	λ' $(\mu\mathrm{m/V})$	C_{ox} $(\mathrm{fF}/\mu\mathrm{m}^2)$	t_{ox} (nm)	L_{ov} $(\mu\mathrm{m})$	$\dfrac{C_{db0}}{W}$ $\left(\dfrac{fF}{\mu\mathrm{m}}\right)$
NMOS	0.4	240	0.05	8.5	4	0.04	0.3
PMOS	-0.4	60	-0.05	8.5	4	0.02	0.3